Lecture 1: *Basic Concepts*

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Outline

- Basic concepts
- Precision specifications of instr.
- Summary

Contents

- ❒ Error
- ❒ Accuracy
- ❒ Precision
- ❒ Trueness

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Definition of Errors

- **Error**
  - Let x_0 denote the true value of a variable of interest, and x_i the recorded value. The measurement error is the difference between the recorded and true value

$$\Delta_i = x_i - x_0 \quad i = 1, 2, \cdots, n$$

- **Characteristics of error**
  - Error ≠ 0, always
  - Δ_i are different
  - Error is unpredictable
- **True value**

True Value

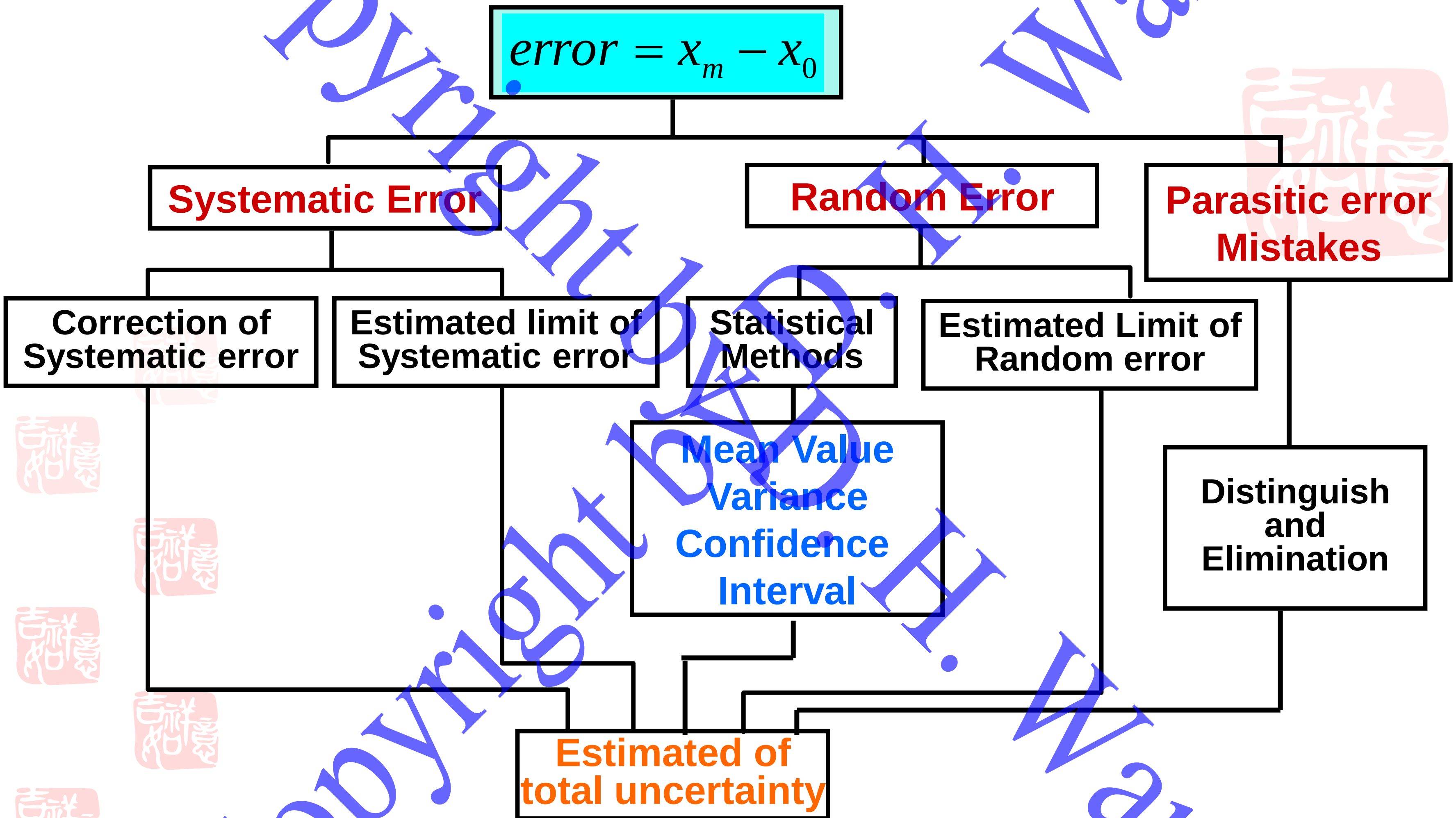
- Reference, accepted value ("True value")
- True value serves as an agreed-on reference for comparison, and which is derived as
  - theoretical or established value, based on scientific principles
  - an assigned or certified value, based on experimental work of some national or international organization
  - consensus or certified value, based on collaborative experimental work under the auspices of a scientific or engineering group

$$1\text{m} = 1650763.73\lambda$$

Kr86的 (2p10—5d5) 能级跃迁在真空中的辐射波长

  - when these are not available, the expectation of the measurable quantity

Types of Measurement Errors



Types of Errors

Errors
- Static parameters' error
- Dynamic parameters' error

Errors
- Independent error
- Relative error

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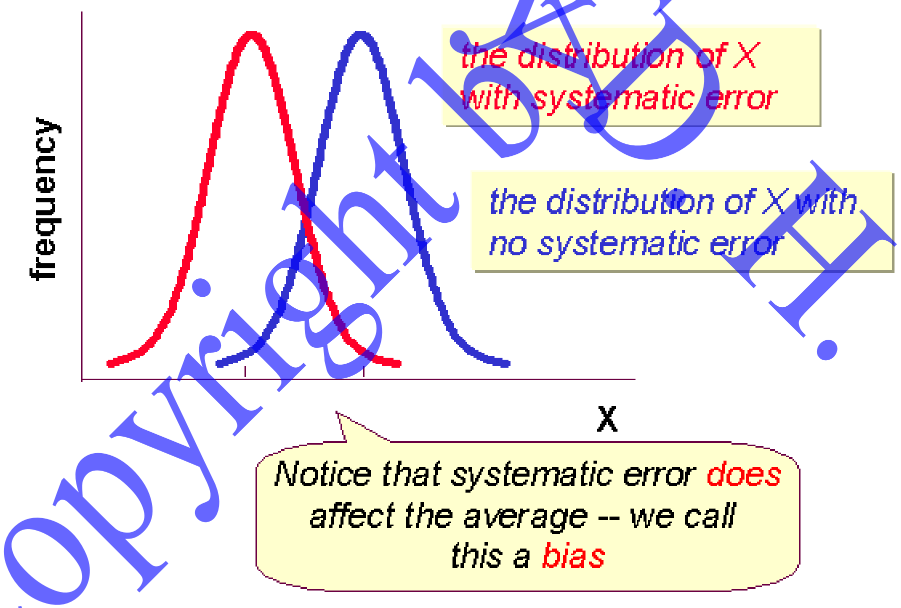
Systematic Error (1/3)

- Systematic error is also called bias.
- The error has the same magnitude and sign in every repeated measurement.
- If the error is really know, correction can be applied and the error can be eliminated.



Systematic Errors (2/3)

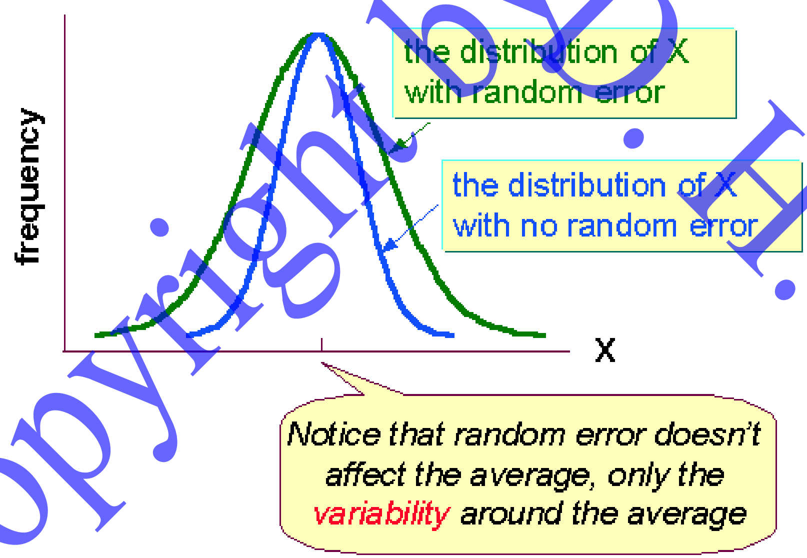
- Source of the systematic errors
  - Internal resistance of an instrument, capacitance or resistance of the connecting wires, temperature, etc.
- Systematic errors cannot be easily discovered.
  - Question
    - How to discover the systematic error?
- Systematic errors can be discovered by using another instrument.
- Require careful measurement design and the evaluation of the error model.

Systematic Errors (3/3)

- Estimated limit of the systematic error
- *Example 1*
  - Eg. a voltage meter with 20 V range
    - It is impossible to calibrate series and shunt resistance for every range.
    - So, there are errors in every range.
    - Hence, a correction table can be assigned to each range.
- *Example 2*
  - Correction is hard to perform

Random Errors (1/2)

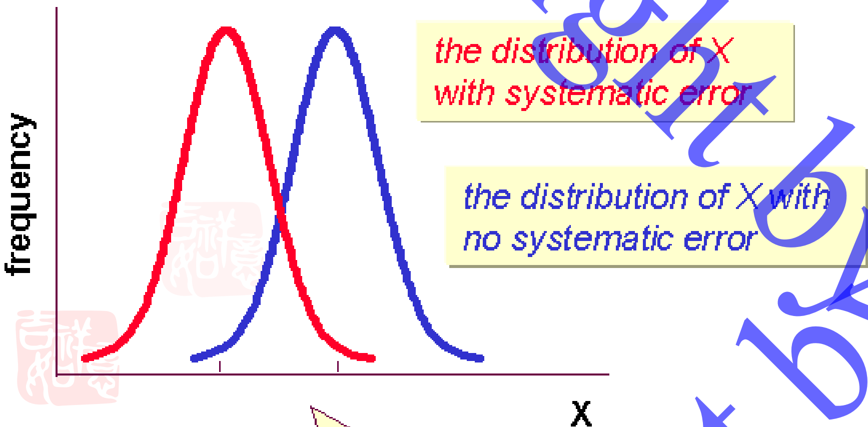
- Random error is caused by any factors that randomly affect measurement of the variable across the sample.
- The important thing about random error is that it does not have any consistent effects across the entire sample. Instead, it pushes observed scores up or down randomly.



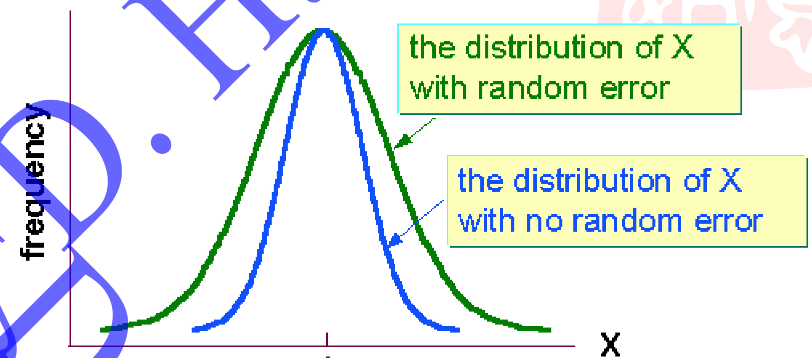
Random Errors (2/2)

- The important property of random error is that it adds variability to the data but does not affect average performance for the group. Because of this, random error is sometimes considered noise.
- Error cannot be correlated with the true value of x(t) and influencing quantity like temperature, magnetic field, etc.
- The error is modeled by a random variable.
- Use estimation theory to retrieve the real measured results.

Systematic Errors and Random Errors



Notice that systematic error does affect the average -- we call this a bias



Notice that random error doesn't affect the average, only the variability around the average

Parasitic Errors or Mistakes

- Typical mistakes include reading the wrong numbers from a measure.
- Mistakes are sometimes called gross errors or blunders.
- Mistakes must be eliminated.
- Question
  - How to discover and eliminate the Mistakes?

Presentation of Measurement Errors

- Absolute error $Error = x_m - x_t = \Delta x$
  - Point estimator $\hat{x}_t = x_m - \Delta x$
  - Interval estimator $\hat{x}_t \in (x_m \pm \Delta x)$
  - eg. error is 0.5 bit, 1 mV, 1 uA, ...
- Relative error in terms of measured value

$$\hat{x}_t = x_m(1-\frac{\Delta x}{x_m}) = x_m(1-\varepsilon_r) \quad or \quad \hat{x}_t \in x_m(1\pm\frac{\Delta x}{x_m}) = x_m(1\pm\varepsilon_r)$$

$$\varepsilon_r = \frac{\Delta x}{x_m}$$

额定相对误差：示值绝对误差与示值的比值。

Relative error is dimensionless

- Relative error in terms of the full scale
  - 1% of the full scale (1.5 V if full scale is 150 V)

引用误差：绝对误差的最大值与仪器示值范围的比值。

引用误差 <= 额定相对误差

Accuracy (1/3)

- Definition: A measure of the capability of an instrument or sensor to faithfully indicate the value of the measured signal.
- Accuracy: The closeness of agreement between a test result and the accepted reference value [ISO 5725].
- The accuracy is often specified as:

  Accuracy = error (in % of reading) + offset

- The offset error of an instrument is defined as the output that will exist when it should be zero or, alternatively, the difference between the actual output value and the specified output value under some particular set of conditions.

Accuracy (2/3)

- Ex: a 5 ½ digit voltmeter can have an accuracy of

  0.0125% of reading + 24 uV

  on its 2.5 V range, which results in an error of 149 uV when measuring a 1 V signal.

- On the other hand, the resolution of this same voltmeter is 12 uV, or 12 times better than the accuracy.

  the accuracy ≥ 12 x the resolution

Accuracy (3/3)

- Accuracy is NOT related to resolution; however, the accuracy level can never be better than the resolution of the instrument.
- It is important to note that the accuracy of an instrument depends not only on the instrument, but on the type of signal being measured.

Precision (1/2)

- Definition: The measure of the stability of an instrument and its capability to give the same measurement over and over again for the same input signal.
- Precision: The closeness of agreement between independent test results obtained under stipulated conditions [ISO 5725].
- Precision is generally considered to consist of a short-term (repeatability) and long-term (reproducibility) component, as given by:

Precision = 1- |Xn - Av(Xn)|/|Av(Xn)|
where Xn = the value of the nth measurement
Av(Xn)=the average value of the set of n measurements

Precision (2/2)

- Ex: If you are monitoring a constant voltage of 1 V, and you notice that your measured value changes by 20 mV between measurements, for example, then your measurement precision is

$$\text{Precision} = (1 - 20\text{ mV}/1\text{V}) \times 100 = 99.998\%$$

This specification is most valuable when you are using the voltmeter to calibrate a device, or when you are performing relative measurements.

Trueness

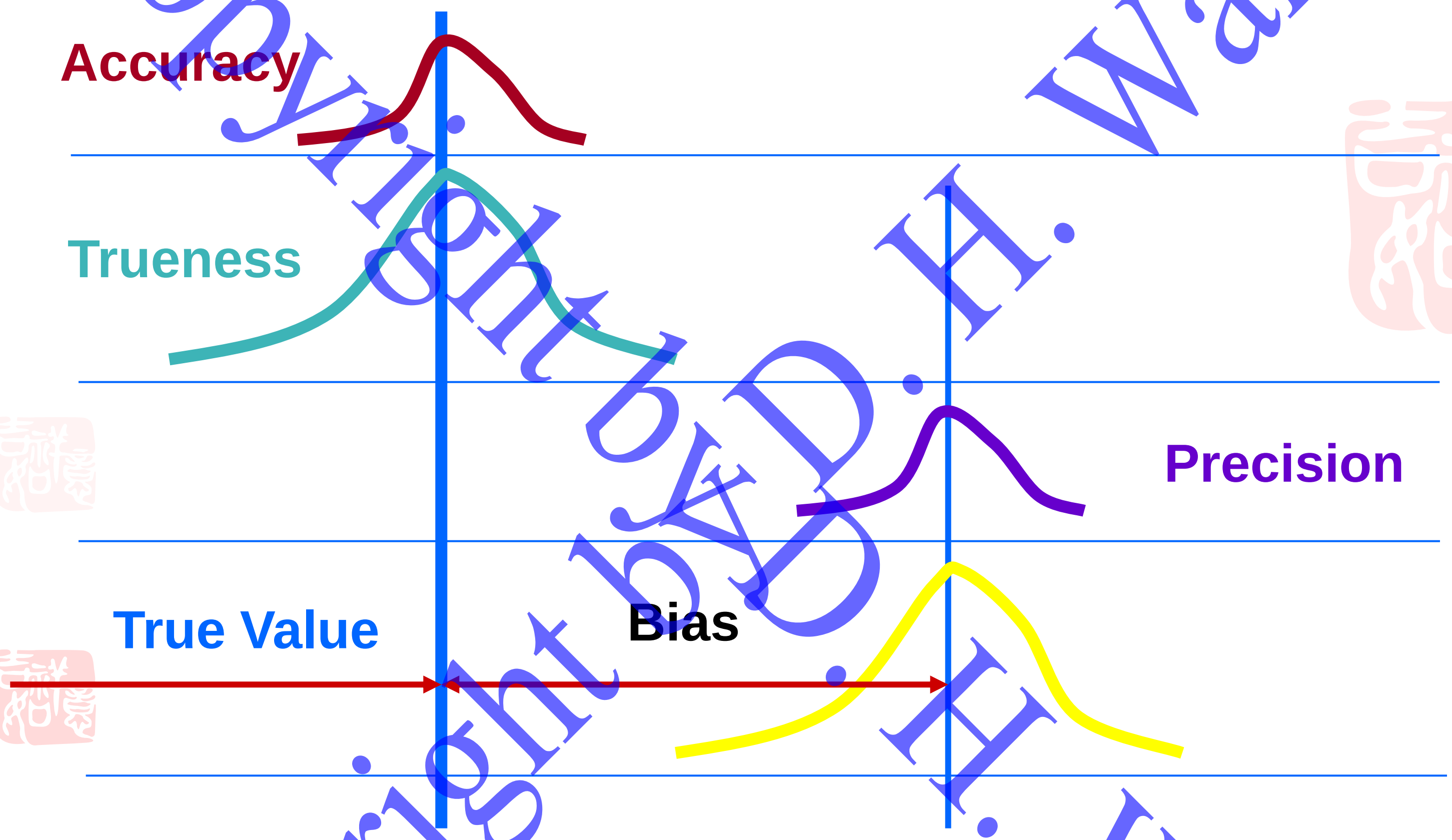
❒ Definition

- the closeness of agreement between the average value obtained from a large series of test results and an accepted reference value [ISO 5725].
- The measure of trueness is usually expressed in terms of bias.

❒ Bias

- The difference between the expectation of the test results and an accepted reference value [ISO 5725].

Accuracy, Precision and Trueness



- Accuracy is the ability to tell the truth.
- Precision is the ability to tell the same story each time.

The END

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Categories

- Static characteristics
- Dynamic characteristics

Static Characteristics

- ❒ **Definition**
- ❒ **Precision specifications**
  - Linearity
  - Indicate error and Repeatability
  - Sensitivity
  - Resolution/discrimination
  - Stability and zero draft
  - Hysteresis

Static Characteristics (1/2)

- **Definition: the relationship between the output of an instrument and its relative input when the input does not or slowly vary with the time.**

$$y = f(x) = \left(a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^n\right) x + K$$

x, y **-- input, output**

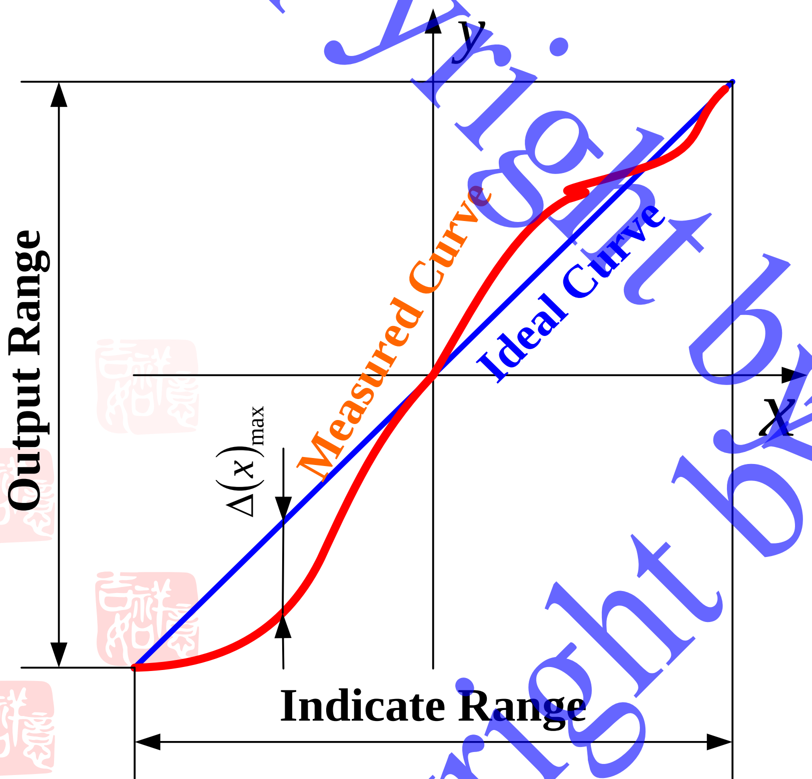
$a_0, a_1, a_2, \cdots, a_n$ **-- coefficients**

- **When** $a_1 = 0, a_2 = 0, \cdots, a_n = 0$

$$y = a_0 x + K$$

Linear relationship

Static Characteristics (2/2)



- The ideal relationship

$$y = a_0 x$$

- Nonlinear error

$$\Delta(x) = f(x) - (a_0 x + K)$$

Linearity

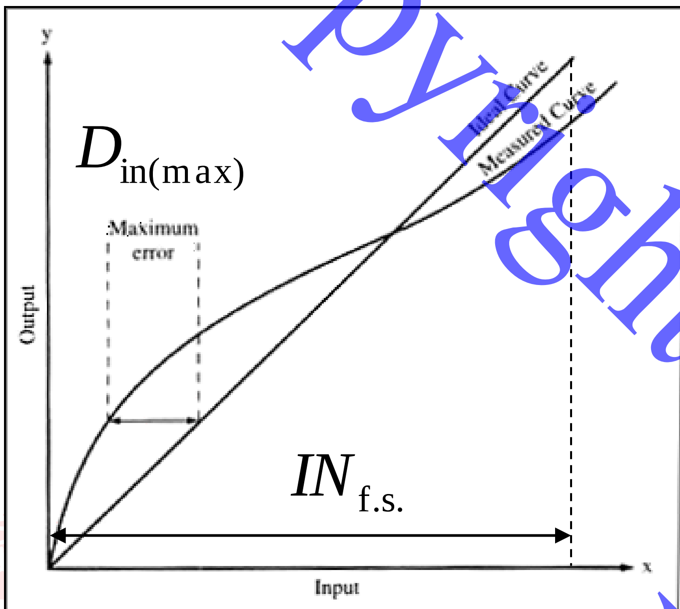
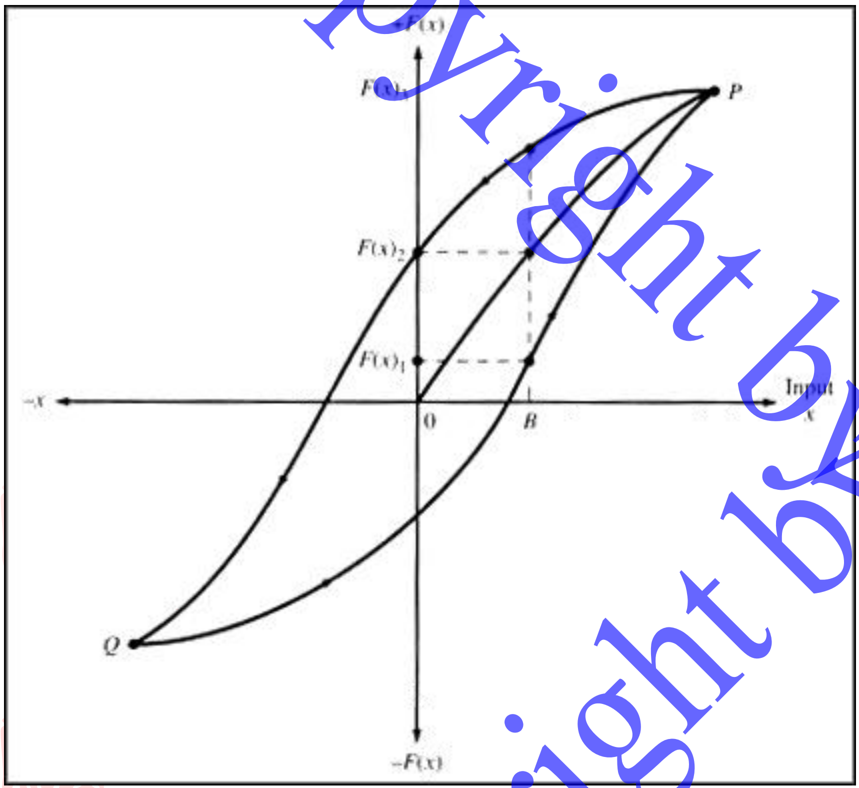


Figure shows a somewhat exaggerated relationship between the ideal, or least squares fit, line and the actual measured or *calibration* line

- **Definition: The linearity of the instrument is an expression of the extent to which the actual measured curve of an instrument departs from the ideal curve.**
- **Linearity is often specified in terms of percentage of nonlinearity, which is defined as:**

$$Nonlinearity\ (\%) = \frac{D_{in(max)}}{IN_{f.s.}} \times 100$$

Hysteresis (1/2)



Hysteresis curve

- **Definition**
  - The change in the value measured by an instrument or a device,
  - when the direction of the applied signal is changed.

Hysteresis (2/2)

- Ex: the fuel gage of a car may show a lower value when the tank is filled to half level, than when the tank is gradually emptied to half level due to driving for a period of time.
- An instrument should be capable of following the changes of the input parameter regardless of which direction the change is made, hysteresis is the measure of this property.
- Hysteresis due to looseness in mechanical joint is also called backlash.

Repeatability (1/3)

- Definition: The ability of an instrument to measure the same value under repeated identical conditions over a short period of time.
  - Repeatability is the short-term component of accuracy.
  - The long-term component of accuracy is called reproducibility.
- Repeatability: precision under repeatability conditions.
- Repeatability conditions - where independent test results are obtained with the same method on identical test items in the same laboratory by the same operator using the same equipment within short intervals of time [ISO 5725].

Repeatability (2/3)

- When ascertaining the repeatability of a measurement, the following conditions should be evaluated and documented
  - What physical principle is used for the measurement
  - The actual measurement technique
  - Methods of observation of the measurement
    - by human being, computer, or some combination hereof
  - Measuring system, including cabling, interconnection, special software algorithms, and so on
  - ...

Repeatability (3/3)

- **When ascertaining the repeatability of a measurement, the following conditions should be evaluated and documented**
  - **…**
  - **Traceability of reference standards**
  - **Physical location of measurement, including any potential environmental influences, such as temperature, humidity, magnetic field, barometric pressure, and so on**
  - **Actual time of the measurement**

Reproducibility

- Definition
  - The ability of an instrument or sensor to measure the same value over a long period of time. See also repeatability, which is the ability to measure the same value over a short period.
- Reproducibility
  - precision under reproducibility conditions.
- Reproducibility conditions: where test results are obtained with the same method on identical test items in different laboratories with different operators using different equipment.

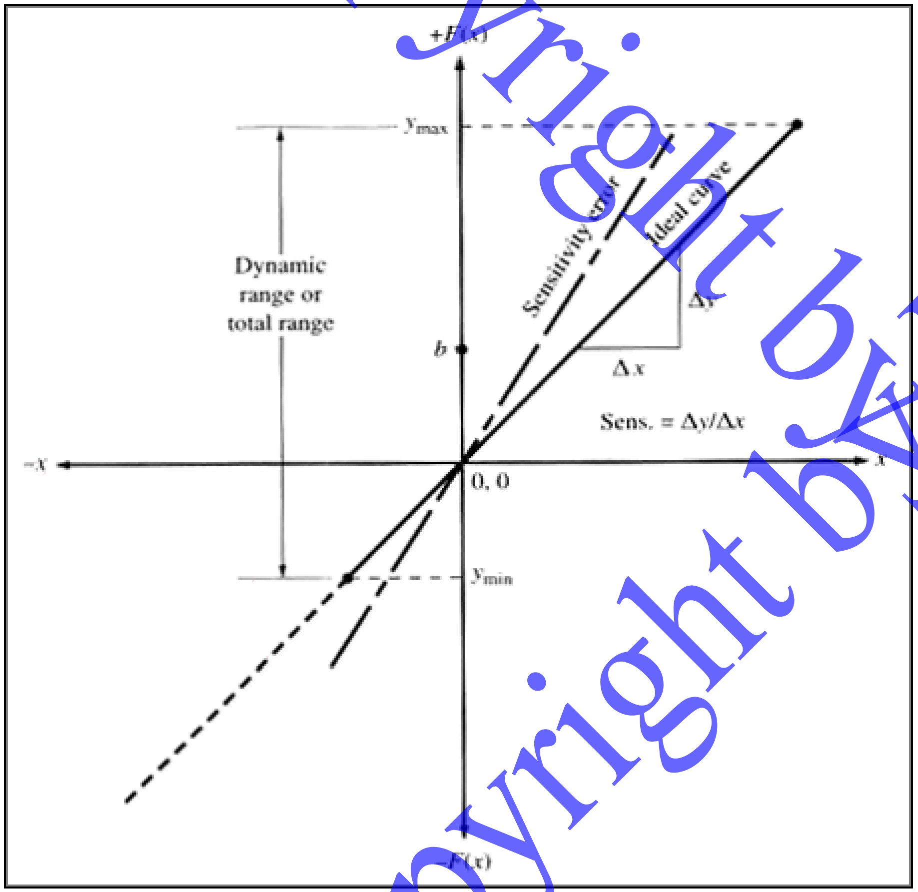
Resolution and Discrimination (1/2)

- Definition
  - The smallest amount of input signal change that an instrument or sensor can detect.
- The term, discrimination, is also used for resolution.
- Resolution can be expressed either as a proportion of the reading (or the full-scale reading) or in absolute terms.
- Resolution is determined by the instrument noise (either circuit or quantization noise) and the smallest change that is detectable by the display system of the instrument.
- Ex: …

Resolution and Discrimination (2/2)

- Ex: if you have a noiseless voltmeter that has 5 ½-displayed digits and is set to the 20 V input range, the resolution of this voltmeter is 100 uV.
  - This can be determined looking at the change associated with the least significant digit.
  - Now, if this same voltmeter had 10 counts of peak-to-peak noise, then the effective resolution is decreased to 1 mV, because any signal change less then 1 mV is indistinguishable from the noise.

Sensitivity (1/2)



❒ Definition

- The sensitivity of an instrument is defined as the slope of the output characteristic curve (ΔY/ΔX) or, more generally, the minimum input of physical parameter that will create a detectable output change.

Sensitivity (2/2)

- Usually, sensitivity is defined at the lowest range setting of the instrument.
- Ex: an AC meter with a lowest measurement range of 10 V may be able to measure signals with 1 mV resolution but the smallest detectable voltage it can measure may be 15 mV.
  - In this case, the AC meter has a resolution of 1 mV but a sensitivity of 15 mV.

Stability and Drift

- **Stability**
  - the ability of a measuring instrument to maintain constant its metrological characteristics with time.
- **Drift**
  - A change in reading or value that occurs over long periods. Changes in ambient temperature, component aging, contamination, humidity and line voltage may contribute to drift.

The End

Thank you very much for your attention !

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Dynamic Characteristics

- ❒ Definition
- ❒ Precision specifications
  - Dynamic deviation error
  - Dynamic repeatability error
  - Frequency response accuracy
- ❒ Mathematical model
- ❒ Zero-, First-, Second-Order Instruments
- ❒ Unit Impulse Response Functions

Dynamic Characteristics

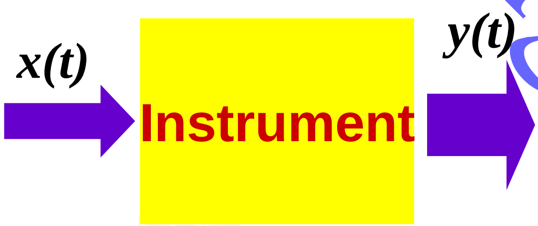
- ❒ **Dynamic characteristics**
  - **the relationship between the output of an instrument and its relative input when the input does vary with the time.**
- ❒ **Precision specifications**
  - **Dynamic deviation error**
  - **Dynamic repeatability error**
- ❒ **Ideal instruments and frequency response accuracy**

Mathematical Model

- Generalized mathematical model with differential equation
- Transfer Function
- Unit impulse response function
- Frequency response function

Mathematical Model

❒ **Generalized mathematical model of instruments**



$$a_n \frac{d^n y}{dt^n} + a_{n-1} \frac{d^{n-1} y}{dt^{n-1}} + \cdots a_1 \frac{dy}{dt} + a_0 y = b_m \frac{d^m x}{dt^m} + b_{m-1} \frac{d^{m-1} x}{dt^{m-1}} + \cdots b_1 \frac{dx}{dt} + b_0 x$$

$y \equiv$ Output quantity

$x \equiv$ Input quantity

$t \equiv$ time

$a's, b's \equiv$ Combinations of system physical parameters, assumed constant

Mathematical Model

❒ **Generalized mathematical model of instruments**

$$\left(a_n D^n + a_{n-1} D^{n-1} + \cdots a_1 D + a_0\right) y$$
$$= \left(b_m D^m + b_{m-1} D^{m-1} + \cdots b_1 D + b_0\right) x$$

The differential operator $D \equiv \mathrm{d}/\mathrm{d}t$

Zero-Order Instrument--Model

$$a_n \frac{d^n y}{dt^n} + a_{n-1} \frac{d^{n-1} y}{dt^{n-1}} + \cdots a_1 \frac{dy}{dt} + a_0 y = b_m \frac{d^m x}{dt^m} + b_{m-1} \frac{d^{m-1} x}{dt^{m-1}} + \cdots b_1 \frac{dx}{dt} + b_0 x$$

$$\Rightarrow \quad a_0 y = b_0 x$$

- **Definition**
  - Any instrument or system that closely obeys the above equation over its intended range of operating conditions is defined to be a Zero-Order Instrument
- **Static sensitivity (or steady-state "gain")**

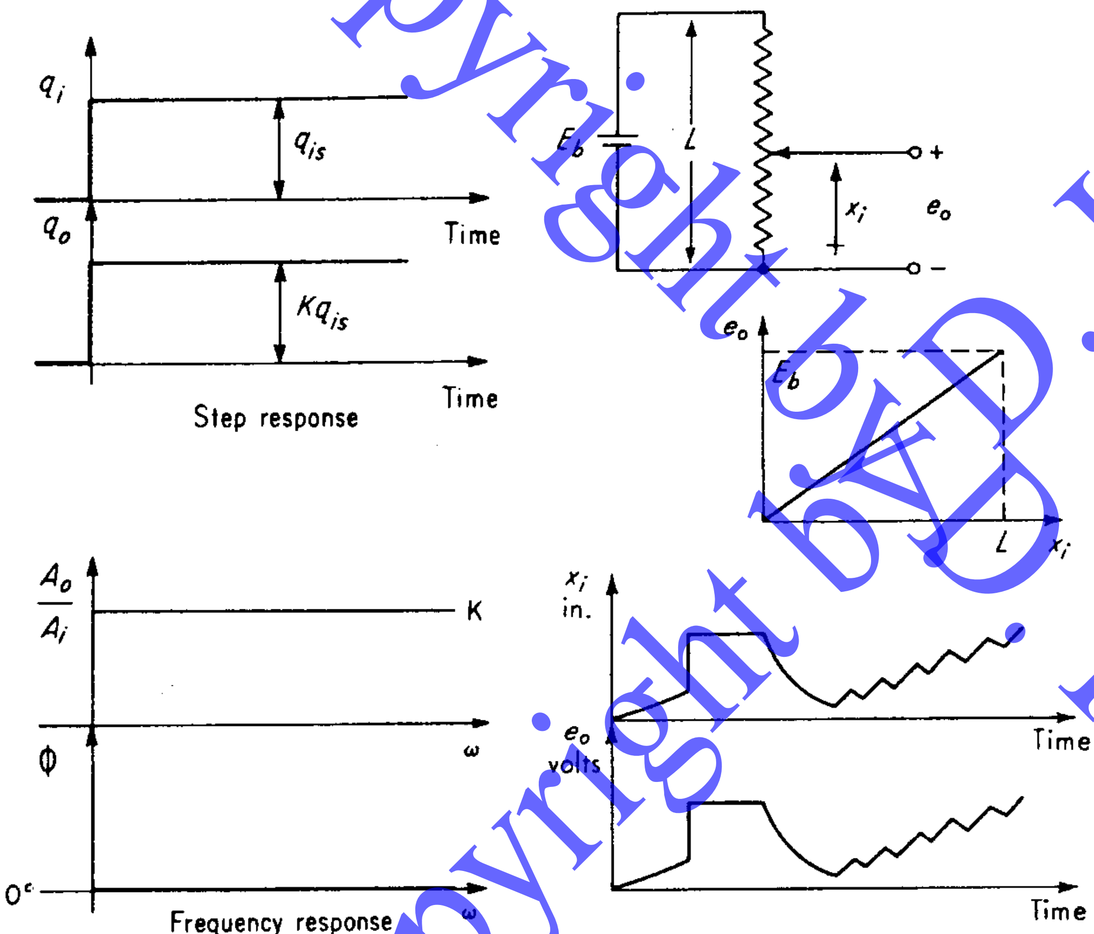
$$y = \frac{b_0}{a_0} x = Kx \qquad K \equiv \frac{b_0}{a_0} \equiv static\ sensitivity$$

Zero-Order Instrument--Model

❒ Note

- No matter how x might vary with time, the instrument output (reading) follows it perfectly with no distortion or time lag of any sort
- The zero-order instrument represents ideal or perfect dynamic performance and is thus a standard against which less perfect instruments may be compared

Zero-Order Instrument--Example



- ❒ A displacement-measuring potentiometer

$$e_o = \frac{x_i}{L} E_b = K x_i \qquad K = \frac{E_b}{L}$$

- ❒ If you examine this measuring device more critically, you will find that it is not exactly a zero-order instrument.

First-Order Instrument--Model

$$a_n \frac{d^n y}{dt^n} + a_{n-1} \frac{d^{n-1} y}{dt^{n-1}} + \cdots a_1 \frac{dy}{dt} + a_0 y = b_m \frac{d^m x}{dt^m} + b_{m-1} \frac{d^{m-1} x}{dt^{m-1}} + \cdots b_1 \frac{dx}{dt} + b_0 x$$

$$\Rightarrow \quad a_1 \frac{dy}{dt} + a_0 y = b_0 x$$

- **Definition**
  - **Any instrument that follows this equation is, by definition, a first-order instrument.**
- **In mathematics, a first-order equation has the general form**

$$a_1 \frac{dy}{dt} + a_0 y = \left(b_m D^m + b_{m-1} D^{m-1} + \cdots b_1 D + b_0\right) x$$

First-Order Instrument--Model

$$a_1 \frac{dy}{dt} + a_0 y = b_0 x \quad \Rightarrow \quad \frac{a_1}{a_0}\frac{dy}{dt} + y = \frac{b_0}{a_0} x$$

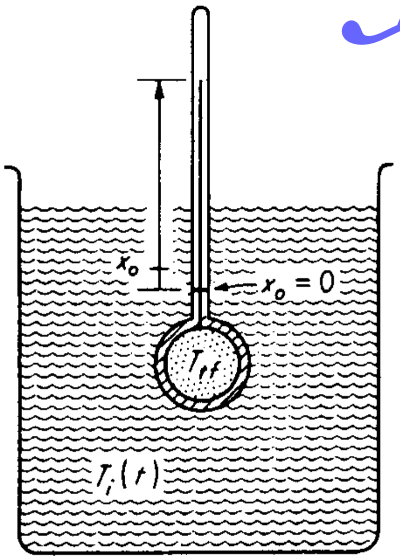
$$\Rightarrow \quad (\tau D + 1) y = Kx$$

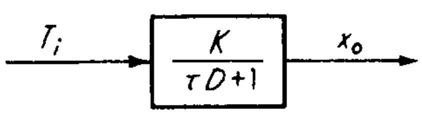
$$K \equiv \frac{b_0}{a_0} \equiv \text{static sensitivity}$$

$$\tau \equiv \frac{b_0}{a_0} \equiv \text{time constant}$$

The operational transfer function $$\frac{y}{x}(D) = \frac{K}{\tau D + 1}$$

First-Order Instrument--Example

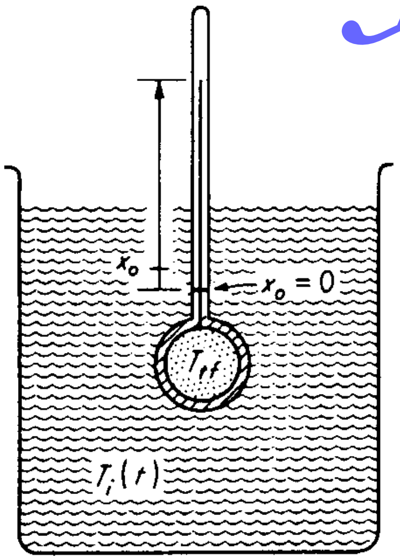


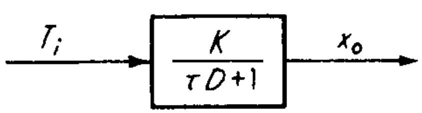


- A liquid-in-glass thermometer
- The principle of operation of such a thermometer is the thermal expansion of the filling fluid which drives the liquid column up or down in response to temperature changes.
- Assumptions
  - the temperature Ti(t) is uniform throughout the fluid at any given time.
  - the mechanical lag is negligible compared with the thermal lag.

First-Order Instrument--Example

- A liquid-in-glass thermometer
- Modeling





$$x_0 = \frac{K_{ex} V_b}{A_c} T_{tf}$$

$x_0 \equiv$ displacement from reference mark, m

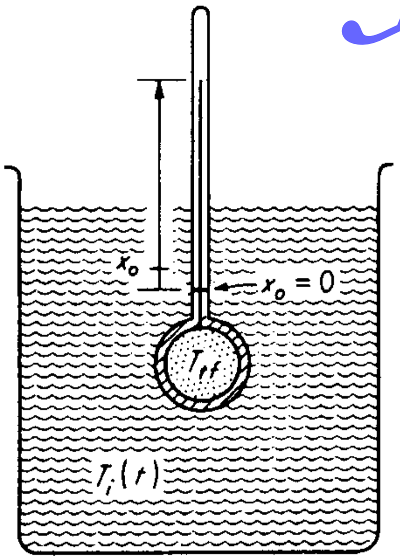
$T_{tf} \equiv$ temperature of fluid in bulb (assumed uniform throughout bulb volume), $T_{tf} = 0$ when $x_0 = 0$, ^oC

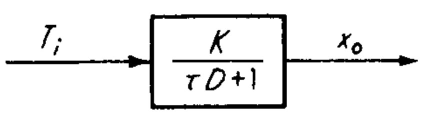
$K_{ex} \equiv$ differential expansion coefficient of thermometer fluid and bulb glass, $m^3/(m^3.C^o)$

$V_b \equiv$ volume of bulb, m^3

$A_c \equiv$ cross-sectional area of capillary tube, m^3

First-Order Instrument--Example





- A liquid-in-glass thermometer
- Modeling

$$\text{Heat in} - \text{heat out} = \text{energy stored}$$

$$UA_b\left(T_i - T_{tf}\right)dt - 0(\text{assume no heat loss}) = V_b \rho C dT_{tf}$$

$U \equiv$ overall heat transfer coefficient across bulb wall, $W/(m^2 \cdot {}^\circ C$

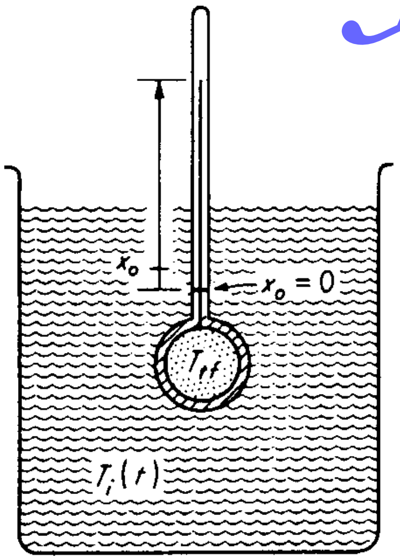
$A_b \equiv$ heat transfer area of bulb wall, m^2

$\rho \equiv$ mass density of thermometer fluid, $\mathrm{kg/m^3}$

$C \equiv$ specific heat of thermometer fluid, $\mathrm{J/(kg \cdot {}^\circ C)}$

First-Order Instrument--Example

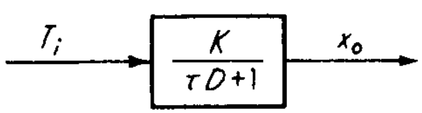
- A liquid-in-glass thermometer
- Modeling



$$UA_b(T_i - T_{tf})dt - 0(\text{assume no heat loss}) = V_b\rho C dT_{tf}$$

$$V_b\rho C\frac{dT_{tf}}{dt} + UA_bT_{tf} - = UA_bT_i$$

$$x_0 = \frac{K_{ex}V_b}{A_c}T_{tf}$$



$$\frac{\rho CA_c}{K_{ex}}\frac{dx_o}{dt} + \frac{UA_bA_c}{K_{ex}V_b}x_o = UA_bT_i$$

$$K \equiv \frac{K_{ex}V_b}{A_c} \qquad \tau \equiv \frac{\rho CV_b}{UA_b}$$

Second-Order Instrument--Model

$$a_n\frac{d^n y}{dt^n}+a_{n-1}\frac{d^{n-1}y}{dt^{n-1}}+\cdots a_1\frac{dy}{dt}+a_0 y$$
$$=b_m\frac{d^m x}{dt^m}+b_{m-1}\frac{d^{m-1}x}{dt^{m-1}}+\cdots b_1\frac{dx}{dt}+b_0 x$$

$$\Rightarrow\quad a_2\frac{d^2 y}{dt^2}+a_1\frac{dy}{dt}+a_0 y=b_0 x$$

$$K\equiv\frac{b_0}{a_0}\equiv\text{static sensitivity}$$

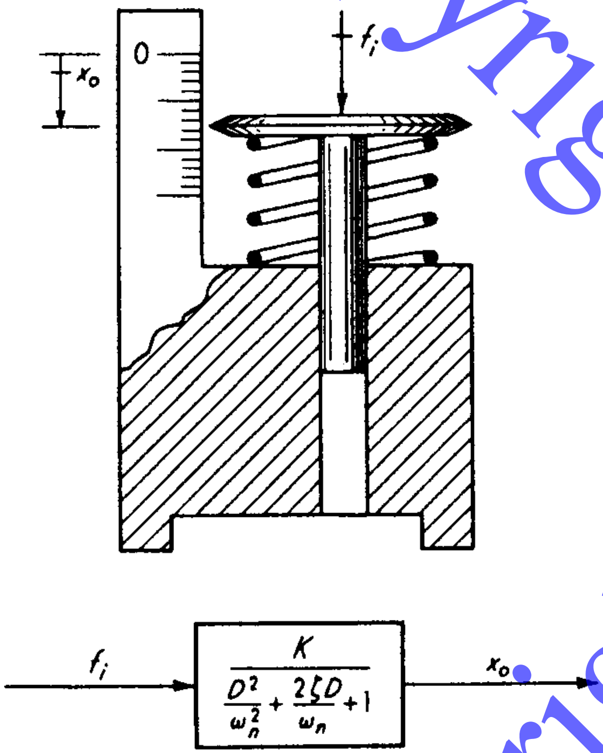
$$\omega_n\equiv\sqrt{\frac{a_0}{a_2}}\equiv\text{undamped natural frequency,rad/time}$$

$$\zeta\equiv\frac{a_1}{2\sqrt{a_0 a_2}}\equiv\text{damping ratio, dimensionless}$$

The operational transfer function

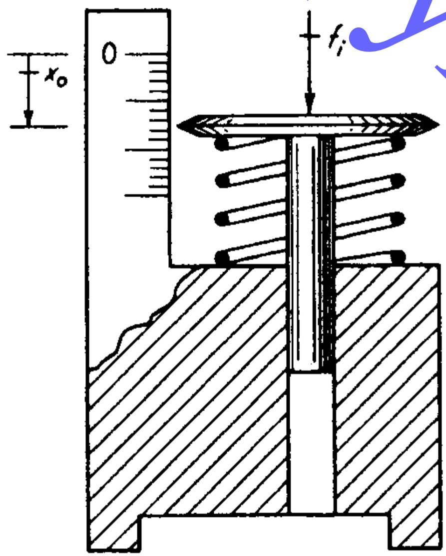
$$\frac{q_0}{q_i}(D)\equiv\frac{K}{\frac{D^2}{\omega_n^2}+\frac{2\zeta D}{\omega_n}+1}$$

Second-Order Instrument--Ex.

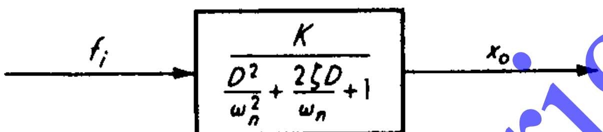


- A force-measuring spring scale
- Assumptions
  - The applied force has frequency components only well below the natural frequency of the spring itself
  - The spring is assumed linear with spring constant
  - We assume perfect film lubrication and therefore a viscous damping effect with constant

Second-Order Instrument--Ex.



- A force-measuring spring scale
- Modeling: The scale can be adjusted so that x0=0 when fi=0 (gravity force will then drop out of the equation), which yields



$$\sum forces = (\text{mass})(\text{acceleration})$$

$$f_i - B\frac{\mathrm{d}x_0}{\mathrm{d}t} - K_s x_0 = M\frac{\mathrm{d}^2 x_0}{\mathrm{d}t^2}$$

$$\left(MD^2 + BD + K_s\right)x_0 = f_i$$

$$K = \frac{1}{K_s}\ \text{m/N} \qquad \omega_n = \sqrt{\frac{K_s}{M}}\ \text{rad/s} \qquad \zeta \equiv \frac{B}{2\sqrt{K_s M}}\ \text{rad/s}$$

Laplace Transform and Transfer Functions

- The analysis of linear systems is facilitated by use of the Laplace Transform.
  - The most important property of the Laplace transform (with zero initial conditions) is the transform of the derivative of a signal.

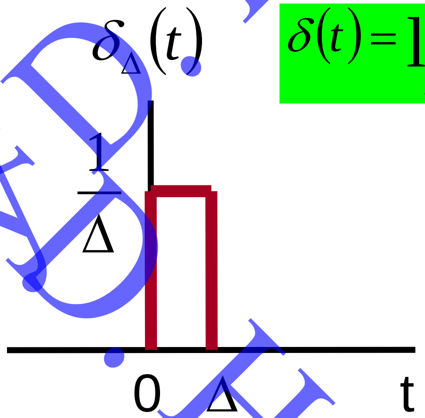
$$L\left\{\dot{f}(t)\right\} = sF(s) \qquad F(s) \overset{\Delta}{=} \int_{-\infty}^{+\infty} f(t)e^{-st}\mathrm{d}t \qquad s = \sigma + j\omega$$

- The relation enables us to find easily the transfer function of a linear continuous system, given the differential equation of that system.

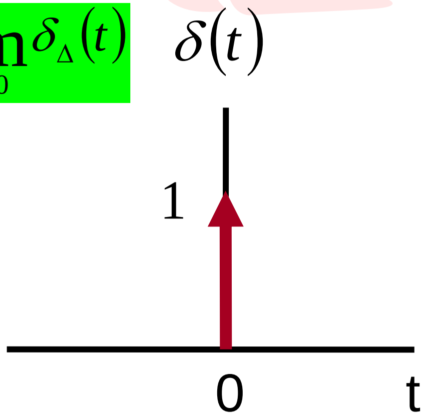
$$H(s) = \frac{Y(s)}{X(s)} = \frac{b_m s^m + b_{m-1}s^{m-1} + \cdots + b_1 s + b_0}{a_n s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0}$$

Unit Impulse Response Functions

- Unit impulse: A short duration signal that goes from 0 to a maximum value, and back to zero again in a very short time. An infinite short impulse is called the Dirac function.



$$\delta(t) = \lim_{\Delta \to 0} \delta_\Delta(t)$$



A tennis racket applies a force of large magnitude that acts on the ball for a very shot period of time

http://www.usopen.org/en_US/news/photos/imagepages/2003-08-11/200308111060040996513.html

- Laplace Transform $L\{\delta(t)\} = 1$

Unit Impulse Response

- If $x(t)=\delta(t)$, then $Y(s)=H(s)$
- Unit impulse response
  - The time domain response of a system to an idealized infinitely short impulse (Dirac Function).

$$h(t)=L^{-1}\{H(s)\}=L^{-1}\{Y(s)\}=y_s(t)$$

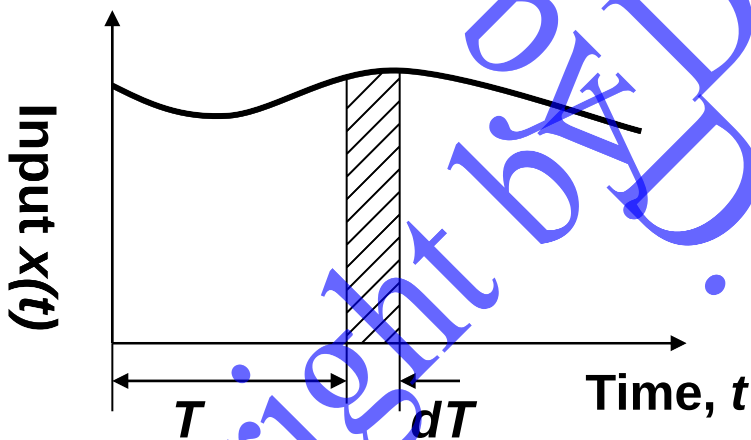
Unit Impulse Response Functions

❒ The convolution integral (the superposition integral)

$$y(t) = h(t) * x(t)$$

$$= \int_0^t h(\tau)x(t-\tau)\mathrm{d}\tau = \int_0^t x(\tau)h(t-\tau)\mathrm{d}\tau$$

Duhamel's Integral



Representation of a general dynamic input by a series of impulse

The Convolution Integral

- It provides a general method for the analysis of the response of a linear system to an arbitrary input.
  - When *x(t)* is a simple mathematical function, closed-form evaluation of the integral is possible
  - in other case, a numeral technique must be used in the evaluation
- It should be noted that in obtaining the convolution integral equation, the system was assumed to be at rest at time t = 0.

Fourier Transform/Frequency Response Functions

- **Fourier transform**

$$F(j\omega) \overset{\Delta}{=} \int_{-\infty}^{+\infty} f(t) e^{-j\omega t} \mathrm{d}t \qquad f(t) \overset{F}{\longleftrightarrow} F(j\omega)$$

- **When $s = j\omega$, the Laplace Transform corresponds to the Fourier transform**

$$F(s)\big|_{s=j\omega} = F\{f(t)\}$$

- **Frequency response function**

$$H(j\omega) = \frac{Y(j\omega)}{X(j\omega)} = \frac{b_m (j\omega)^m + b_{m-1}(j\omega)^{m-1} + \cdots + b_1 (j\omega) + b_0}{a_n (j\omega)^n + a_{n-1}(j\omega)^{n-1} + \cdots + a_1 (j\omega) + a_0}$$

Time/Frequency Domain

❒ Frequency domain advantages

- Time domain convolution => frequency domain multiplication
- Signal and noise can be decomposed into different bands for very high noise rejection.
- Period waves are discrete in time domain.
- Attainable dynamic range (80-100dB) is much larger.
- Slight nonlinearities can be easily detected (harmonics).
- FFT is a very powerful tool, hence, correlation is much faster via frequency domain then directly in time domain.

❒ Time domain advantages

- It's a natural to view time domain signal.
- Recursive methods provide on-line calculation.
- Time varying systems can be modeled easily.
- Time domain methods are not sensitive to signal types while F domain has leakage for nonperiodic signals.
- Certain nonlinearity are easily recognized (slipping, slew rate).

Complex Frequency/Frequency Domain

❒ Complex frequency domain advantages

- The most important property of the Laplace transform (with zero initial conditions) is the transform of the derivative of a signal.
- Include the transient response

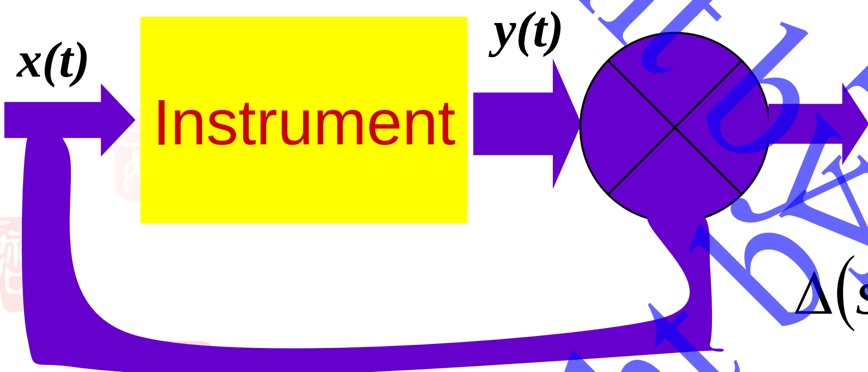
❒ Frequency domain advantages

- Time domain convolution => frequency domain multiplication
- Signal and noise can be decomposed into different bands for very high noise rejection.
- Period waves are discrete in time domain.
- Attainable dynamic range (80-100 dB) is much larger.
- Slight nonlinearities can be easily detected (harmonics).
- FFT is a very powerful tool, hence, correlation is much faster via frequency domain then directly in time domain.
- Don't include the transient response

Precision Specifications

❒ **Dynamic deviation error/动态偏移误差Δ(t):**

✐是一种有规律的或在一定条件下有固定大小和符号的误差，它由输入信号的形式和仪器的动态特性所决定。



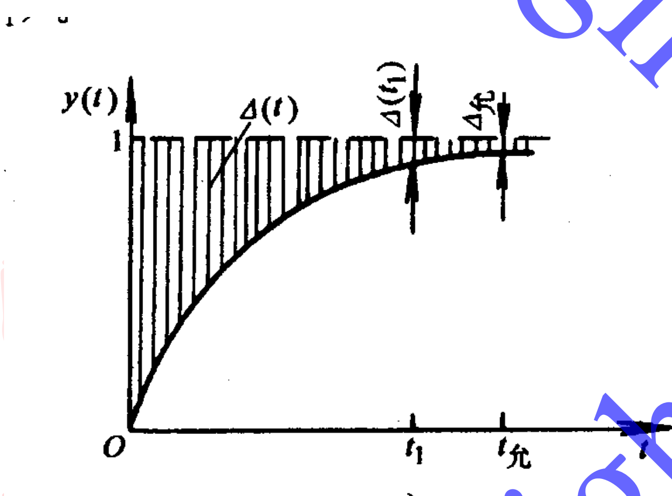
$$\Delta(t) = y(t) - x(t)$$

$$\Delta(s) = Y(s) - X(s) = H(s)X(s) - X(s) = [H(s) - 1]X(s)$$

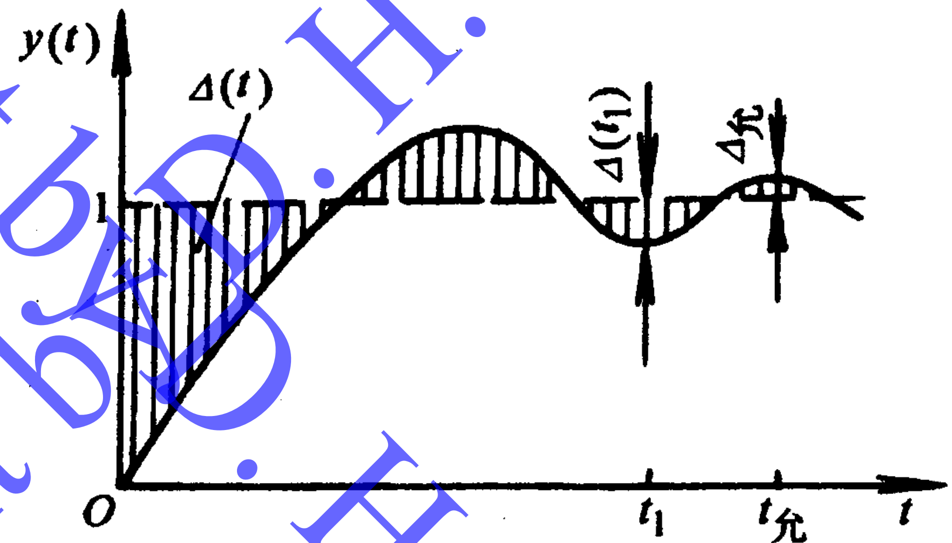
❒ **Dynamic deviation error**
- Steady-state errors
- Transient-state errors

Precision Specifications

❒ Dynamic deviation error under unit step function input



First-order system



Second-order system

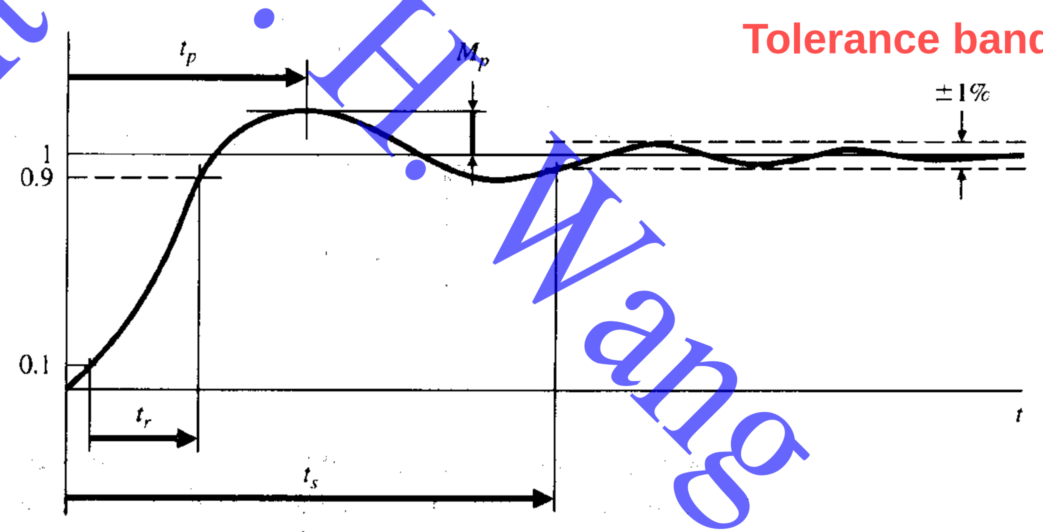
Time-Domain Specifications

- The rise time t_r is the time it takes the system to reach the vicinity of its net set point.
- The settling time t_s is the time it takes the system transient to decay.
- The overshoot M_p is the maximum amount that the system overshoots its final value divided by its final value (and often expressed as a percentage)

$$t_r \approx \frac{1.8}{\omega_n}$$

$$t_s \approx \frac{4.6}{\zeta\omega_n} = \frac{4.6}{\sigma}$$

$$M_p \approx e^{-\pi\zeta/\sqrt{1-\zeta^2}} \quad 0 \le \zeta < 1$$



Dynamic Characteristics

- Dynamic deviation error

$$\Delta(t) = y(t) - x(t)$$

- Methods of analysis the output of instrument
  - Calculating methods
    - Evaluating the output with the theory of linear differential equations
    - Convolution between unit impulse response and input in time domain
    - Inverse Laplace transform of the product of transfer function and input signal
  - Experimental method

Dynamic Characteristics

- Dynamic deviation error
- Methods of Analysis the output of instrument
  - Calculating methods
    - …
  - Experimental method
    - 也可用实验测试的方法得到输出信号 y(t) 的样本集合，在特定的动态测量条件下，通过多次测量，把 y(t) 的均值 M[y(t)] 与被测量信号 x(t) 之差作为测量仪器的动态偏移误差，即

$$\Delta(t) = \mathrm{M}[y(t)] - x(t)$$

Dynamic Repeatability Errors

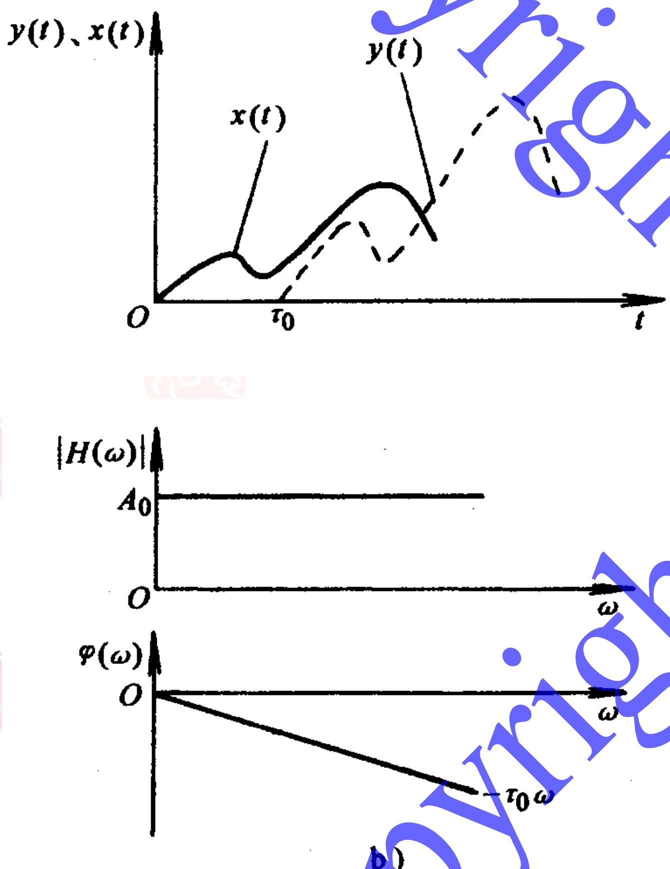
- ❐ 动态重复性误差是指在规定的使用条件下，对同一动态输入信号进行多次重复测量，所测得的各个输出信号在任意时刻 tk 量值的最大变化范围，
  - 通常用三倍的动态输出标准差s(tk)来表示
- ❐ 当输出信号是确定性信号与随机信号的组合时，动态输出的标准差可用下式估计

$$s(t_k)=\sqrt{\frac{1}{n-1}\sum_{i=1}^{n}\left[y_i(t_k)-\bar{y}(t_k)\right]^2}$$

- ❐ 动态重复性误差的变化范围

$$\Delta y(t_k)=\pm 3s(t_k)$$

Characteristics of Ideal Instruments



❐ **Ideal instrument**

$$y(t) = A_0 x(t - \tau_0)$$

$$H(j\omega) = \frac{Y(j\omega)}{X(j\omega)} = A_0 e^{-j\omega\tau_0}$$

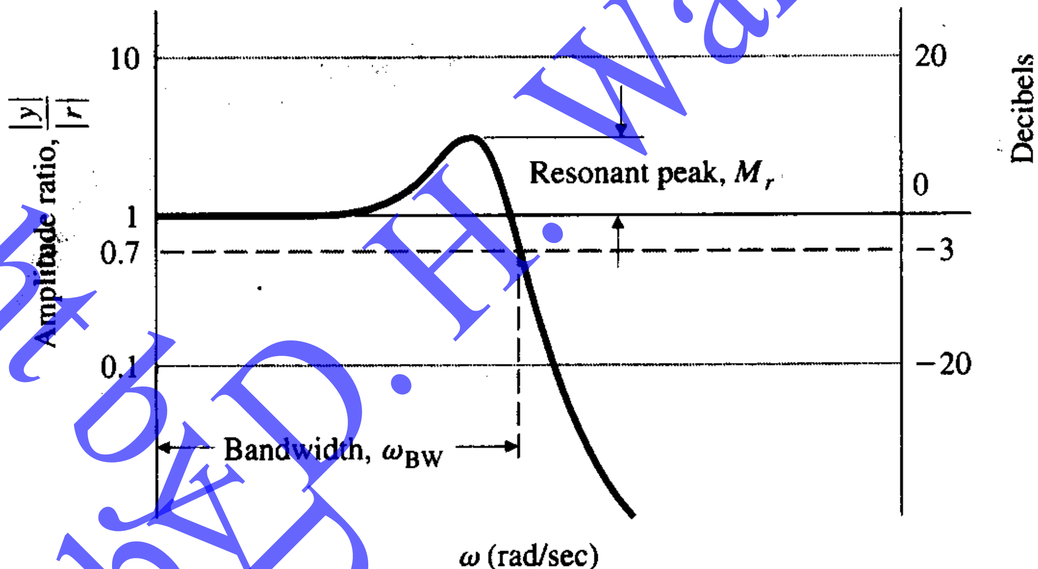
Magnitude property

$$|H(j\omega)| = A_0$$

Phase property

$$\varphi(\omega) = -\omega\tau_0$$

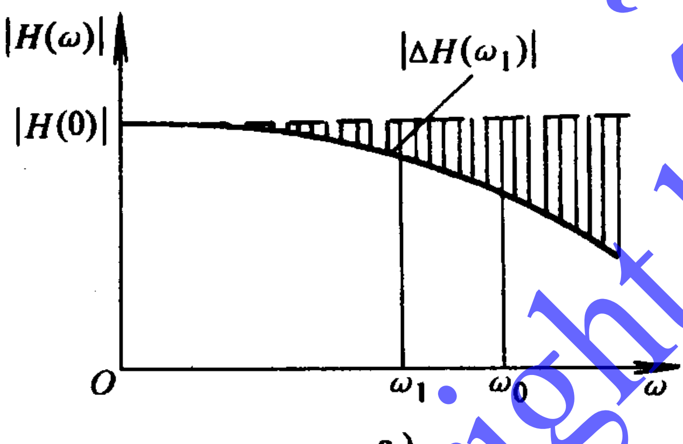
Bandwidth of Instruments

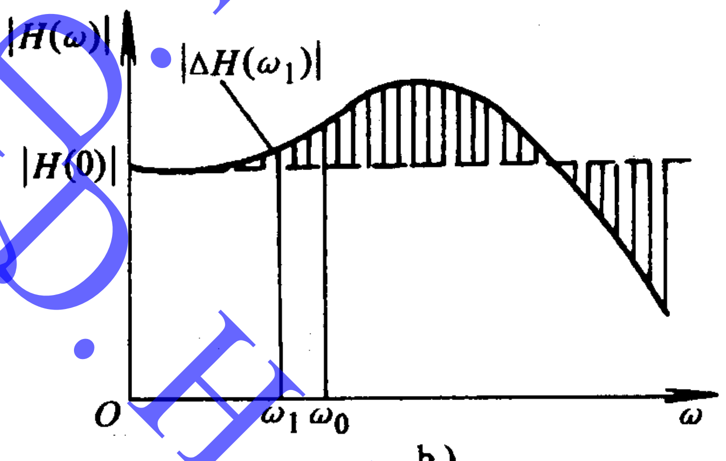


- Bandwith: the maximum frequency at which the output of a system will track an input sinusoid in a satisfactory manner.
- For the instrument, the bandwith is the frequency of *x(t)* at which the output *y(t)* is attenuated to a factor of 0.707 times the input (or down 3 dB)

Frequency Response Accuracy

- ❐ **Maximum magnitude error within the bandwith**
- ❐ **Maximum phase error within the bandwith**
  - Ex: Maximum magnitude error within the bandwith





❖First-order system

❖Second-order system

The END

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Summary (1/3)

- Errors
  - Types of errors
  - Presentation of errors
- Precision, accuracy, and Trueness
- Precision specifications
  - Static characteristics
    - …
  - Dynamic Characteristics
    - …

Summary (2/3)

- Precision specifications
  - Static characteristics
    - Linearity
    - Indicate error and Repeatability
    - Sensitivity
    - Resolution/discrimination
    - Stability and zero draft
    - Hysteresis
  - Dynamic Characteristics
    - …

Summary (3/3)

- Precision specification
  - Static characteristics
    - …
  - Dynamic Characteristics
    - Zero-order instruments
    - First-order instruments
    - Second-order instruments
    - Unit impulse response
    - Transfer function
    - Frequency response function

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The End

Thank you very much for your attention!

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