Lecture 3: *Error Sources & Error Properties of Instruments (Part 2)*

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Outline

- Part 1
  - 仪器误差及其来源
  - Principle errors
- Part 2
  - Manufacturing errors
  - Running errors
- Summary of Lectures 2 and 3

Definition

- ❐ 制造误差
  - 指由仪器的零件、元件、部件和其他各个环节在尺寸、形状、相互位置以及其他参量等方面的制造及装调的不完善所引起的误差。
- ❐ 在仪器误差中，制造误差占有极大的比重。
- ❐ 并不是所有的制造误差对仪器精度都有影响，我们只研究与仪器精度有关的制造误差，又称为原始误差 **(Original error)**。

产生因素

- 由于内外尺寸的配合间隙，对直线运动造成歪斜误差，对回转运动造成径向跳动误差
- 轴与套的圆度会引起轴系的回转误差
- 表面波度和粗糙度会影响运动的平稳性误差
- 导轨导向面直线度误差会引起拖板运动误差
- 在有些差动电路和差动式传感器中，制造中的结构不对称会造成共模误差和零位漂移
- 在光学仪器中，透镜和棱镜的制造误差会引起成像畸变和光线方向的变化

Control Methods

- ❐ 合理地分配误差和确定制造公差
- ❐ 正确应用仪器设计原理和设计原则
- ❐ 合理地确定仪器的结构参数
- ❐ 合理的结构工艺性
- ❐ 设置适当的调整和补偿环节

Definition and Causes

- Definition
  - 仪器使用过程中产生的误差
- Causes
  - Force induced deflection
  - Stress induced deflection
  - Measuring force induced deflection
  - Wear
  - Clearance and backlash induced error
  - Thermal induced error
  - Vibration
  - Interference and environment condition

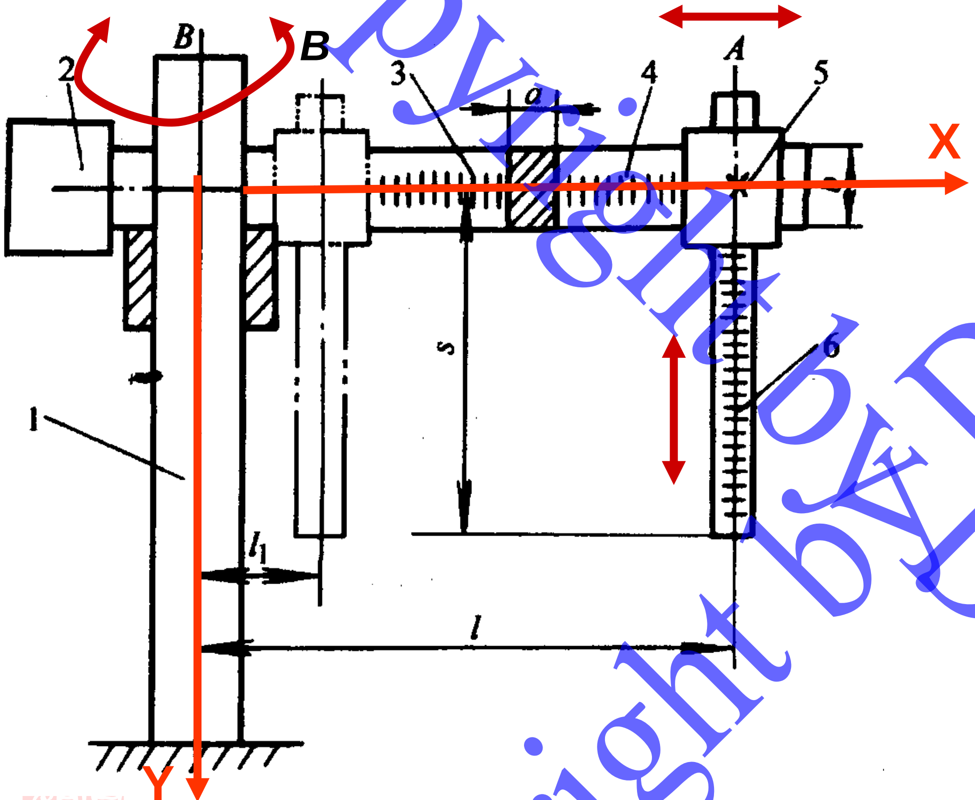
坐标测量机

↗Horizontal-arm CMM

From: http://www.tsk.co.jp/english/

←Bridge-Type CMM

Due to Force Deflections



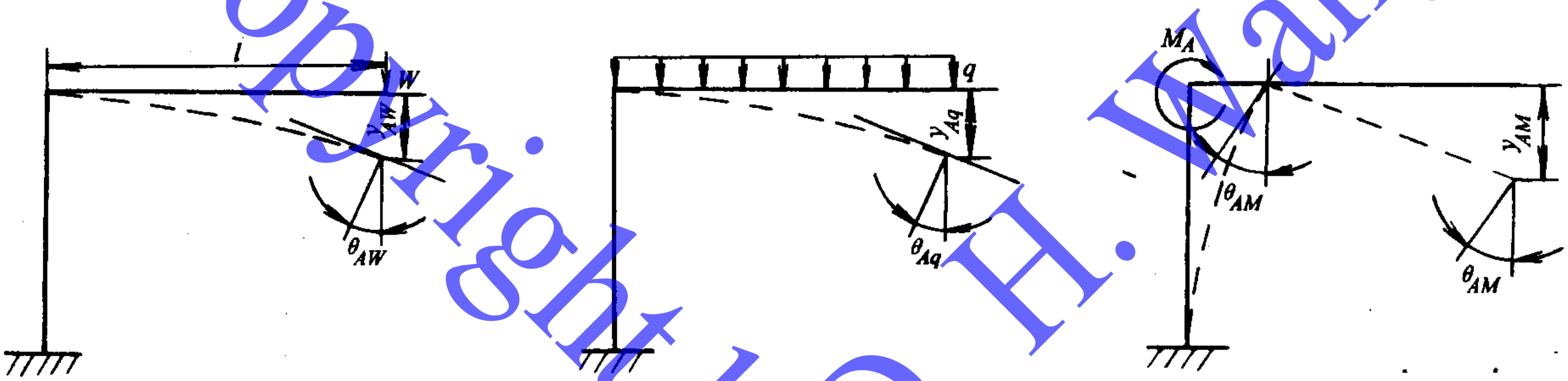
Horizontal-arm CMM

From: http://www.tsk.co.jp/english/

Rotating-arm CMM

$$a \times b = 50mm \times 200mm \quad l = 3000mm \quad W = 200N$$

Load Induced Errors



- 当测头部件位于横臂最外端A处

$$y_A = y_{AW} + y_{Aq} + y_{AM} = -3.56 \text{ mm}$$

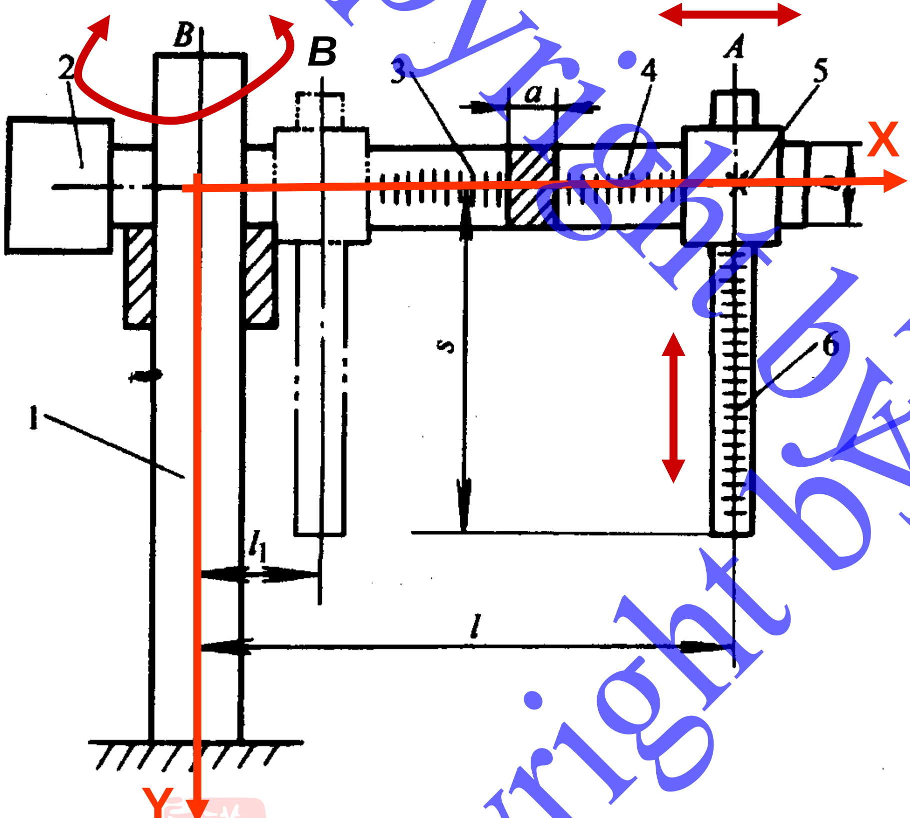
$$\theta_A = \theta_{AW} + \theta_{Aq} + \theta_{AM} = -1.56\times10^{-3} \text{ rad}$$

- 测头部件处在横臂最里端B处

$$y_B = y_{BW} + y_{Bq} + y_{BM} = -0.13 \text{ mm}$$

$$\theta_B = \theta_{BW} + \theta_{Bq} + \theta_{BM} = -0.46\times10^{-3} \text{ rad}$$

Load Induced Errors



- 测头部件从B点移到A点时变形引起的测量误差
  - 在测量方向Y向上引起的测量误差

$$y_A - y_B = 3.43\ \text{mm}$$

  - 在X测量方向引起阿贝误差

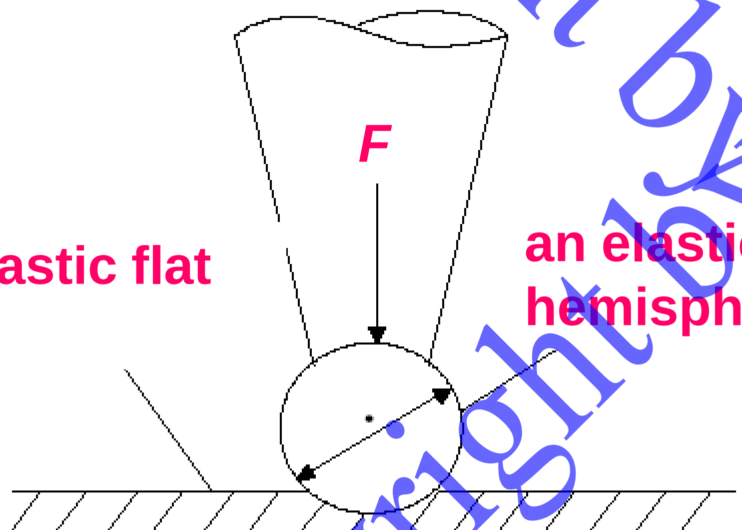
$$s(\theta_A - \theta_B) = 1.11\ \text{mm}$$

Load Induced Errors

- ❐ 对大型仪器，由力变形引起的测量误差相当大
- ❐ 为了减小力变形，在设计过程中要着重
  - 提高仪器结构件的刚度
  - 合理选择支点的位置和材料
  - 适当采用卸荷装置
  - 使重力引起的变形达到最小

Measuring Force Induced Deflections

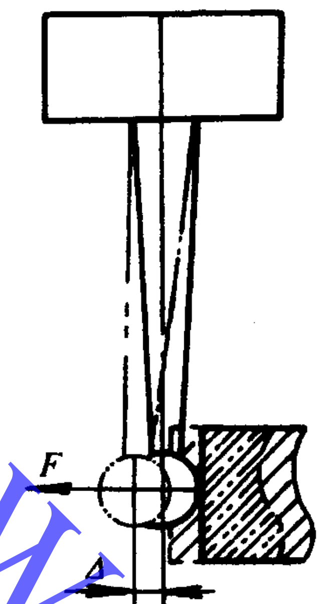
- Deformation error in contact measurement
  - Deformation of the contacting bodies
  - Deflection due to bend



an elastic flat plate

an elastic hemisphere

An elastic hemisphere on an elastic flat plate



Bend

Hertz Contact Theory

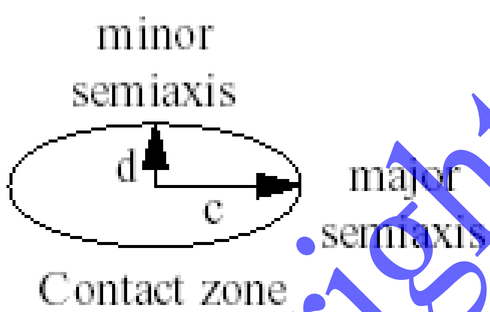
- The theory developed by ***Hertz*** in 1880 remains the foundation for most contact problems encountered in engineering.
- It applies to ***normal contact*** between two elastic solids that are smooth and can be described locally with ***orthogonal radii of curvature*** such as a toroid.
- Further, the size of the actual contact area must be small compared to the dimensions of each body and to the radii of curvature.
- Purpose
- "Exact" solutions for point contact
- Application of Hertz theory

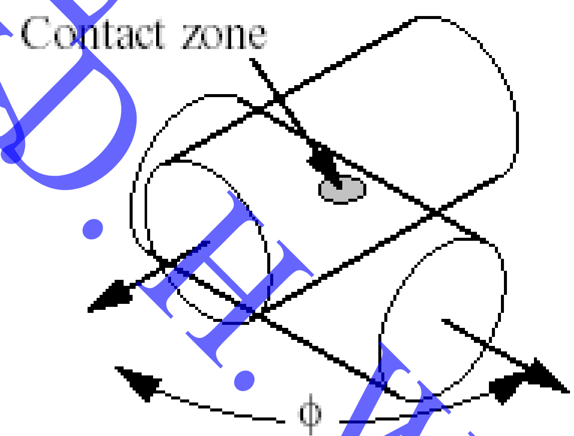
Hertz Contact Theory

- Purpose
  - Describe analysis methods for estimating the stress and deformation that occur when there is point or line contact between two bodies.
- "Exact" solutions for point contact
- Application of Hertz theory

Hertz Contact Theory

- ❒ **"Exact" solutions for point contact**
  - **Hertz theory can be used to accurately determine the stress and deflection in the contact region.**
  - **Contact between curved surfaces results in an elliptical contact region.**





- **The equivalent modulus of elasticity of the system**

Hertz Contact Theory

❒ "Exact" solutions for point contact

- The equivalent modulus of elasticity of the system
  - Based on the elastic modulii and *Poisson's ratio η* of the two materials in contact

$$E_e = \frac{1}{\dfrac{1-\eta_1^2}{E_1} + \dfrac{1-\eta_2^2}{E_2}}$$

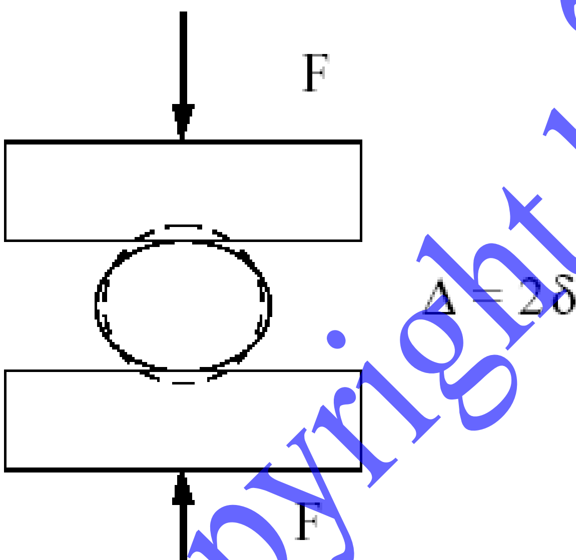
- The equivalent radius or curvature is found

$$R_e = \frac{1}{\dfrac{1}{R_{1_{major}}} + \dfrac{1}{R_{1_{minor}}} + \dfrac{1}{R_{2_{major}}} + \dfrac{1}{R_{2_{minor}}}}$$

Hertz Contact Theory

❐ Application of Hertz theory

- **For contact at a single interface (e.g. an elastic hemisphere on an elastic flat plate), the distance of approach of two far field points in the bodies is:**



$$\delta = \lambda\left(\frac{2F^2}{3R_e E_e^2}\right)^{1/3}$$

Hertz Contact Theory

表4-1 不同接触体在接触力作用下接触区弹性变形量的计算公式

接触面	接触区弹性变形量计算公式
半径为R_1、R_2的两球体相互接触	$\delta = 0.8255\sqrt[3]{(\eta F)^2 \frac{R_2+R_1}{R_1R_2}}$
半径为R_1、R_2的两球体相互接触，两球体弹性模量相同，泊松系数$\mu=0.30$	$\delta = 1.231\sqrt[3]{\left(\frac{F}{E}\right)^2 \frac{R_2+R_1}{R_1R_2}}$
半径为R_1的球体与平面($R_2=\infty$)相互接触	$\delta = 0.8255\sqrt[3]{(\eta F)^2 \frac{1}{R_1}}$
两圆柱体$R_1=R_2=R$轴线正交相互接触，弹性模量相同	$\delta = 0.8255\sqrt[3]{(\eta F)^2 \frac{1}{R}}$

注：1. F——接触力；

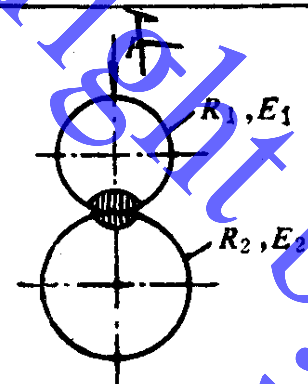
2. $\eta = \frac{1-\mu_1^2}{E_1} + \frac{1-\mu_2^2}{E_2}$，其中$\mu_1$、$\mu_2$、$E_1$、$E_2$为第一、第二物体的泊松系数和弹性模量；

3. 当球体R_2为凹球面时，公式中的R_2+R_1以R_2-R_1替代。

张善锺, *精密仪器精度理论*, 北京: 机械工业出版社, 1993

Hertz Contact Theory

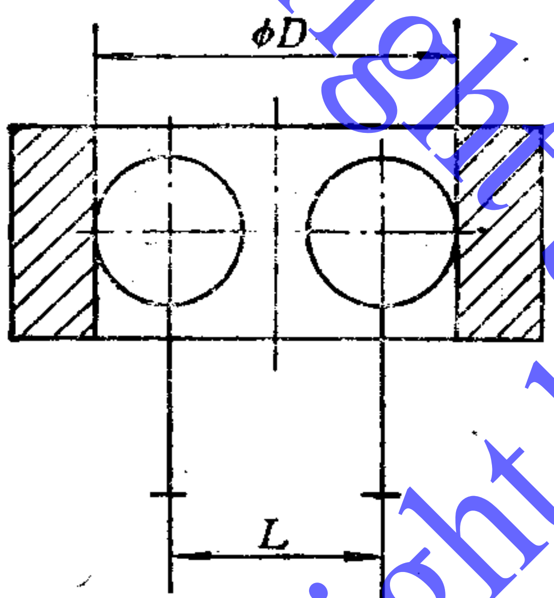
表4-3 接触变形计算公式

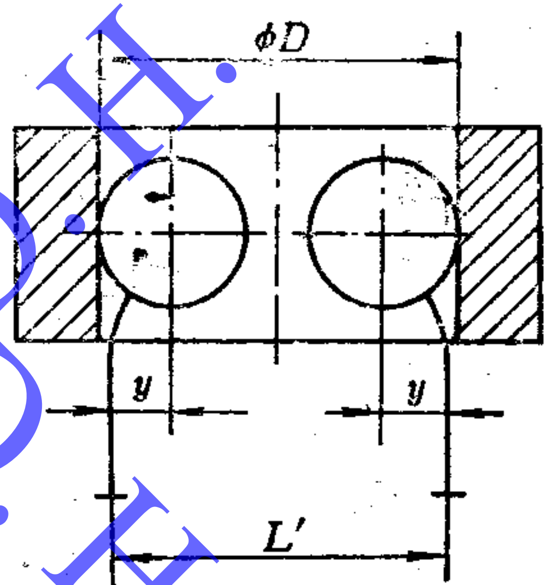
接触情况		接触变形计算公式　F——测量力（单位N）　R——半径（单位m）
球与球		$0.76\sqrt[3]{F^2\left(\frac{1}{E_1}+\frac{1}{E_2}\right)^2\left(\frac{1}{R_1}+\frac{1}{R_2}\right)}$ E_1、E_2——工件与测端材料的弹性模量
球与平面		$0.76\sqrt[3]{\frac{F^2}{R}\left(\frac{1}{E_1}+\frac{1}{E_2}\right)^2}$

陈林才, 张鄂, *精密仪器设计*, 北京: 机械工业出版社, 1991

Measuring Force Induced Deflection

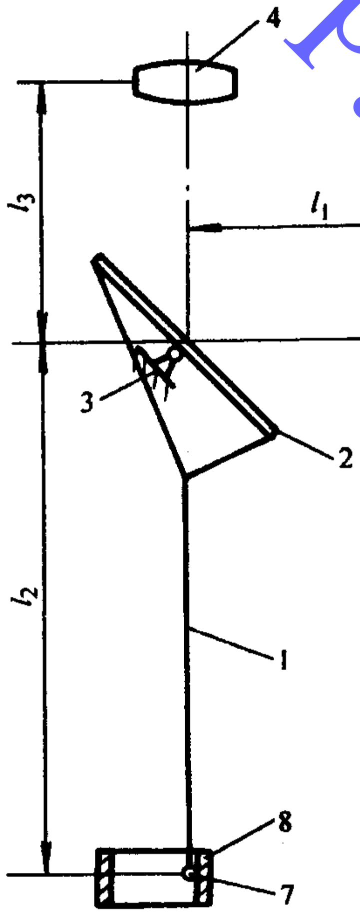
❒ **Deflection due to the rod bending**

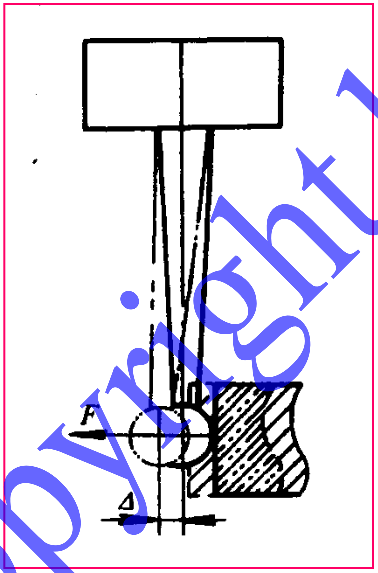




$\Delta D = 0$ $\Delta D = 2y$

Measuring Force Induced Deflection





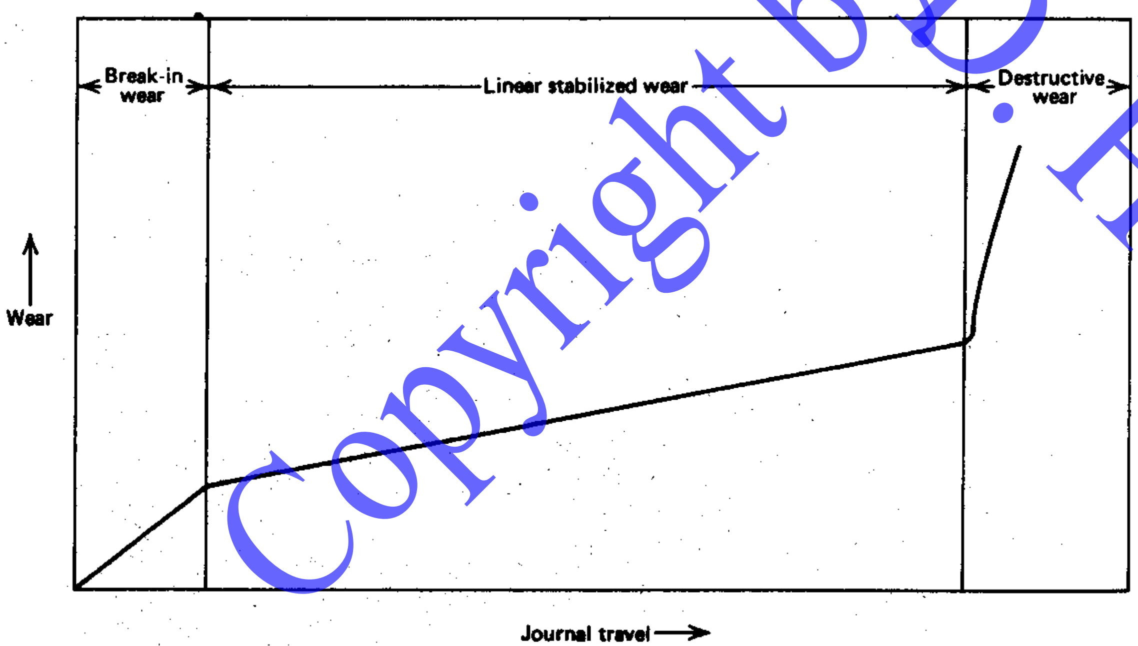
- **Ex:** 在万工显上利用灵敏杠杆测量：设灵敏杠杆长为 **70 mm**，直径约为 **8 mm**，测球直径为 **4 mm**，测杆和被测零件材料同为钢，测量力 **F＝0.2 N**
  - 测球与被测平面之间的接触变形**0.41 um**
  - 测杆的弯曲变形为**0.54 um**
- 接触变形与测杆的弯曲变形对万工显测量精度产生直接的影响

Measuring Force Induced Deflection

- 在测量力作用下接触变形和弯曲变形所引起的运行误差是不可忽视的
- 减小接触变形和弯曲变形影响的措施
  - 接触变形量的大小与接触表面的形状、材料、表面粗糙度以及作用力大小有关
  - 测杆弯曲变形量与测杆的长度和测量力大小有关
  - 在设计中应尽量减小测量力，同时确保测量力恒定
  - 在测量数据处理时进行补偿

Wear

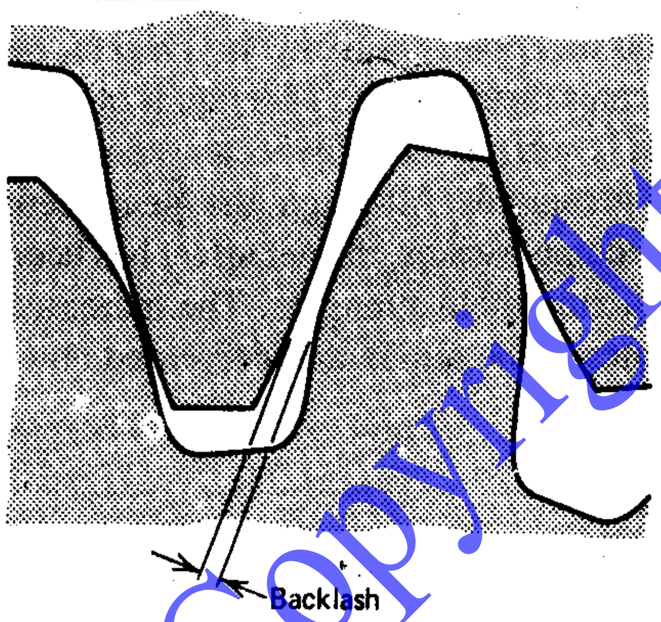
- Types of wear
  - Adhesive wear
  - Abrasive wear due to hard particles
  - Abrasive, hard versus soft materials
  - Pitting of gear tooth caused mainly by rolling contact
- Three stages of wear



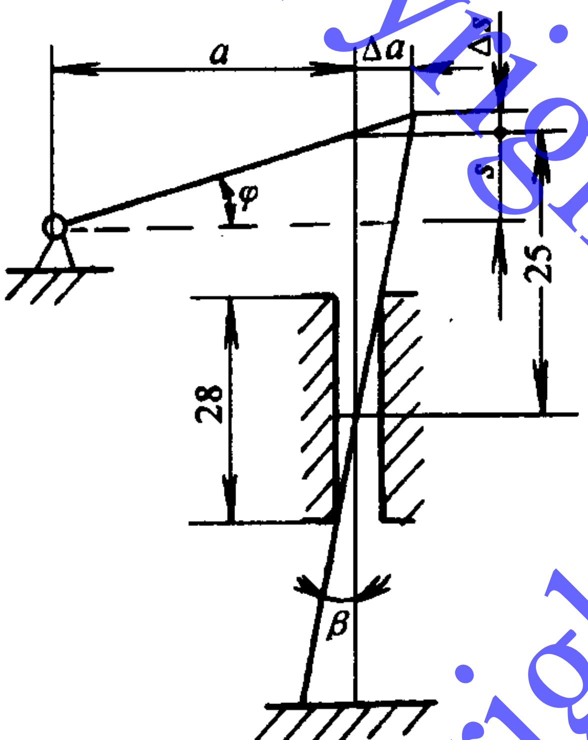
From "Wear in Boundary-Lubricated Bearings," Machine Design, May 16,1974.

Clearance and Backlash

- Backlash occurs only when gears are in mesh.
  - The general purpose of backlash is to prevent gears from jamming together.
  - Backlash also compensate for machining errors and heat expansion.



Clearance and Backlash



- Clearance always occurs when the fit exist.
- The disadvantages of clearance will be discussed in detail in Section titled Error Analysis and accumulation of *Vertical Projection Optimeter.*

Clearance and Backlash

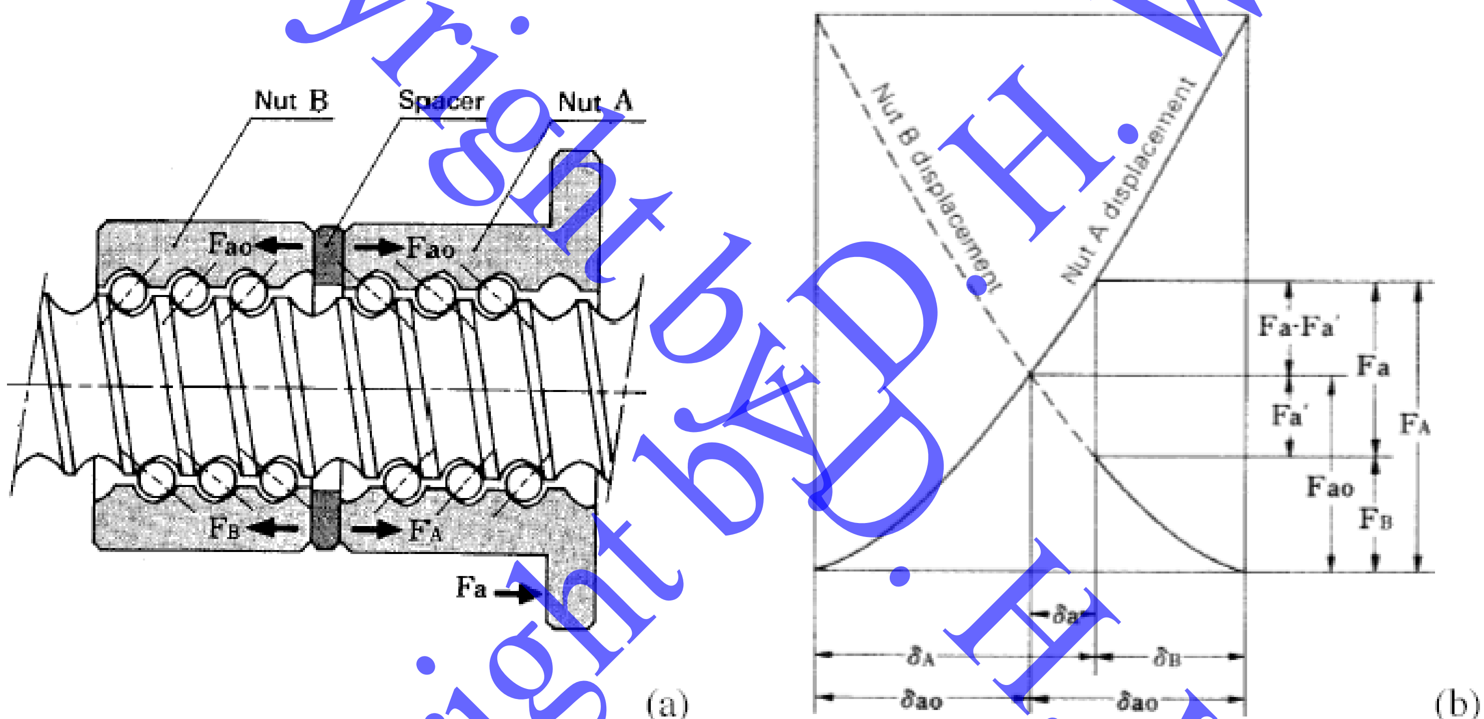
- Backlash mainly exists in ***Servo Mechanisms***
- Friction and backlash are common sources of nonlinearity in a servo mechanism.
- Backlash is usually the more serious problem associated with geared transmissions and lead screws.
- Friction and backlash may lead to
  - limit cycling for systems with output position feedback
  - unacceptable position errors for systems with motor position feedback
  - uneven following error
  - possibly dynamic instability problems
- Computer simulation is a convenient way to test the effects of nonlinearity in a dynamic system.

Anti-Backlash Transmission Design

- ❒ **Preloaded Rolling-Element Bearings**
- ❒ **Preloaded Gear Trains**
- ❒ **Dual-Motor Drives**
- ❒ **Commercial Differential Drives**
  - **Cycloidal Drive（摆线轮驱动）**
  - **Harmonic Drive（谐波驱动）**
  - **Epicyclic Drive（行星轮驱动）**

Anti-Backlash Transmission Design

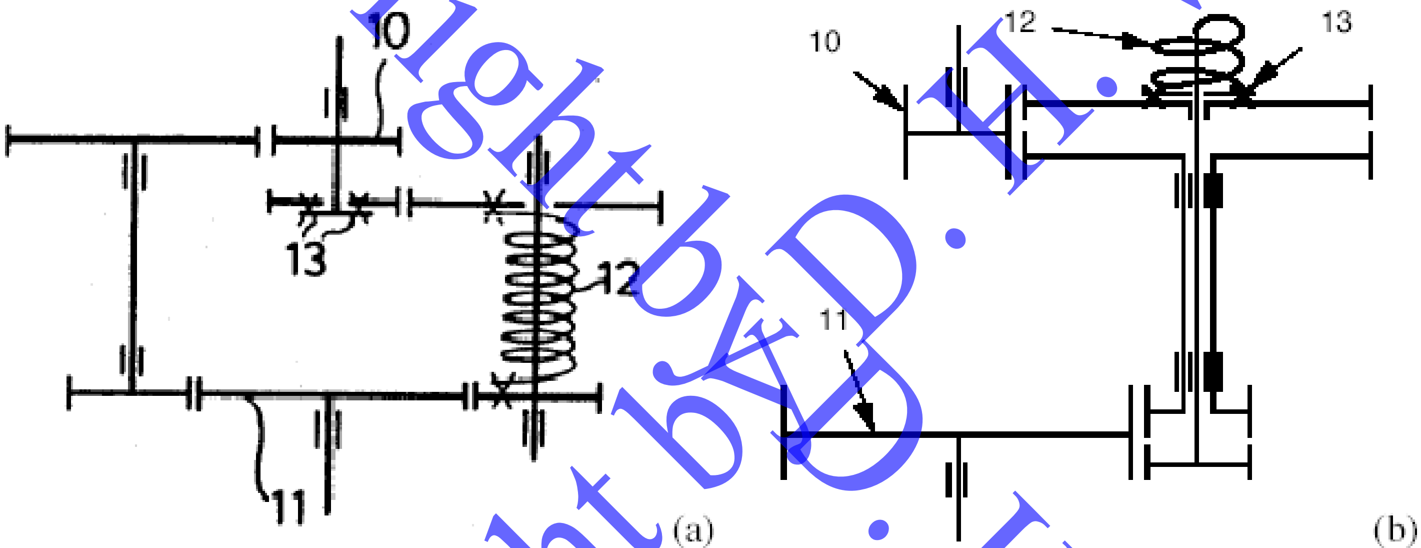
❐ Preloaded Rolling-Element Bearings



A common way to preload a ball screw (a) is to load one nut against the other. The preload diagram (b) represents the forces and displacements that take place between the two nuts.
--Reproduced from a catalog courtesy of THK.

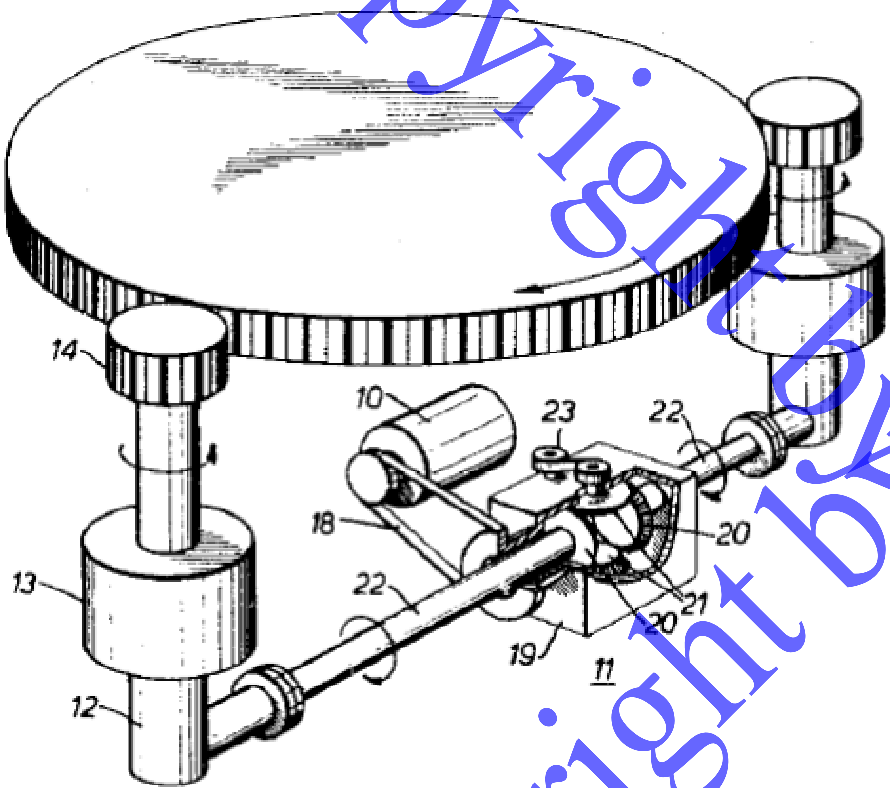
Anti-Backlash Transmission Design

❒ Preloaded Gear Trains



A Type 1 anti-backlash gear train has one stiff path and one path that includes *a compliance element* (Item 12). Part (b) has concentric paths and requires less space. By means of *a preload adjustment device* (Item 13), the paths can be set to oppose one another to eliminate all backlash.

Anti-Backlash Transmission Design

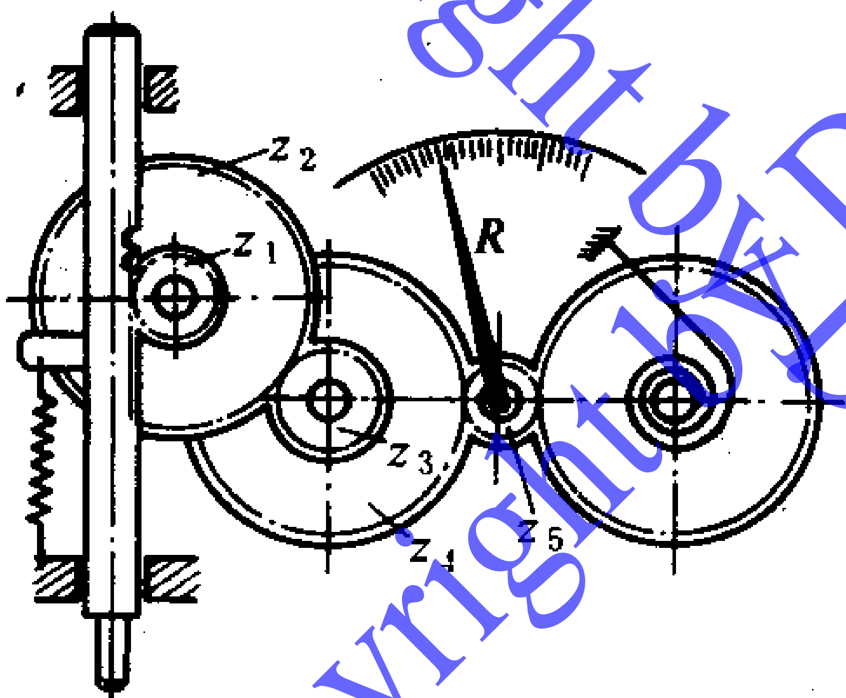


11—a differential; 23—a compliant spring; 20,21—the bevel gears

- **Preloaded Gear Trains (from U.S. Patent 3,665,482)**
  - A Type 2 design for a large tracking antenna [Cresswell, 1972].
  - A Type 2 design has two stiff paths between the input and the output. Compliance is introduced by a preload device usually located at the common input to the two paths.

Anti-Backlash Transmission Design

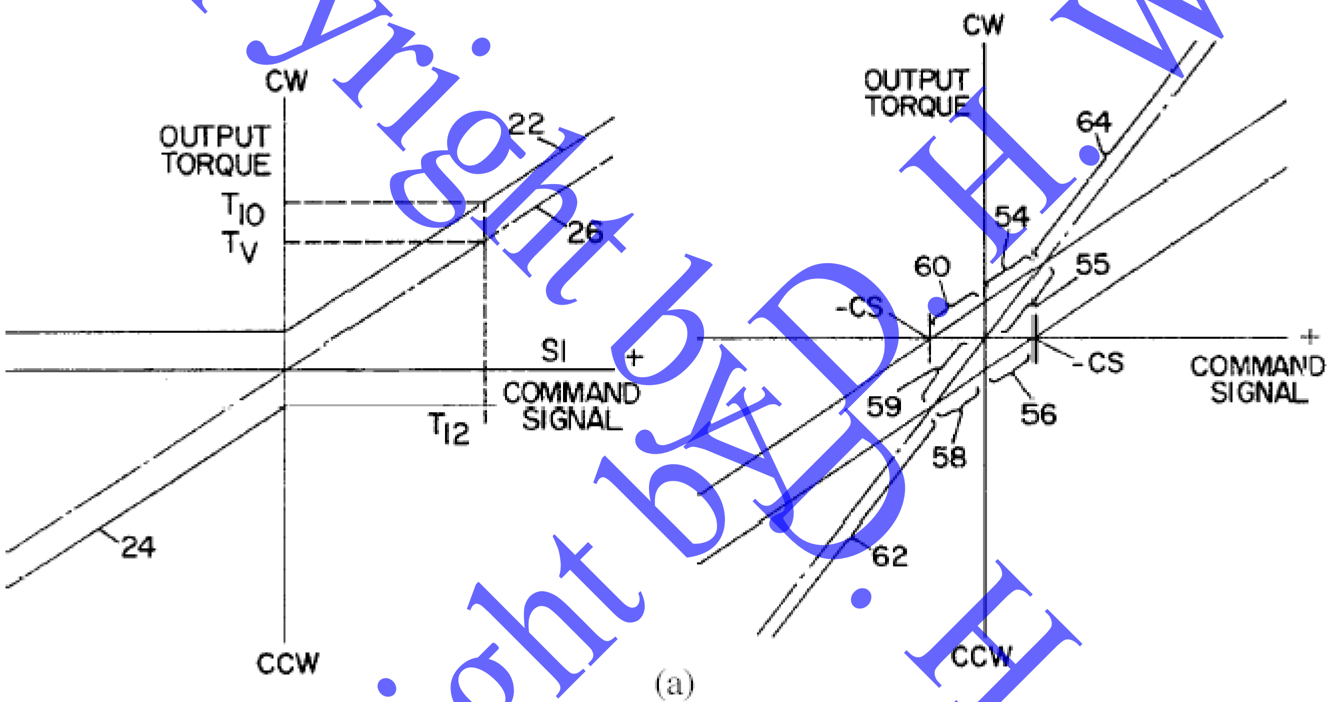
- How to consider the anti-backlash transmission design in *0.001* mm *dial indicator* ?



0.001 mm dial indicator

Anti-Backlash Transmission Design

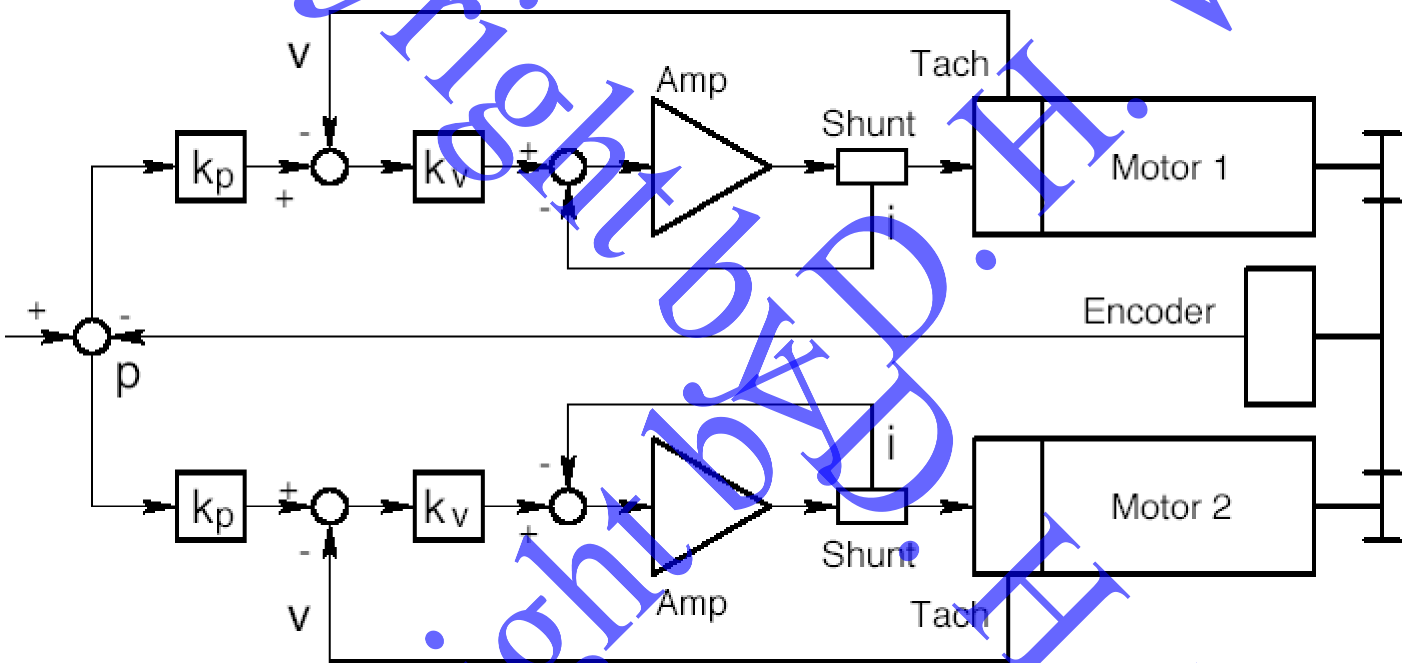
❒ Dual motor drives



The coordination of two drive motors can be very simple. Motor torques (solid lines) add to give a net output torque (center line).
In (a), each motor applies an opposite unidirectional torque.
In (b), the motors have a constant bias torque between them (from U.S. Patent 3,833,847).

Anti-Backlash Transmission Design

❒ **Dual motor drives**



The two transmission paths are preloaded through the control system rather than a mechanical preload device. A disadvantage is the additional motor heating to generate the preload.

Anti-Backlash Transmission Design

- **The principle of differential motion**
  - **Using involute gears arranged differentially, the device outputs the difference between two slightly different gear meshes operating at the speed of the input shaft. Since the gears also operate under output-sized loads, losses can be overwhelming.**
  - **A transmission ratio greater than 50:1 is easy to achieve but the efficiency is typically below 50% making a single-stage device impossible to back drive.**
  - **The differential drives achieve higher efficiency through clever ways to reduce friction. Backlash control for these devices usually comes through control of tolerances and selective fitting.**
- **Commercial differential drives**

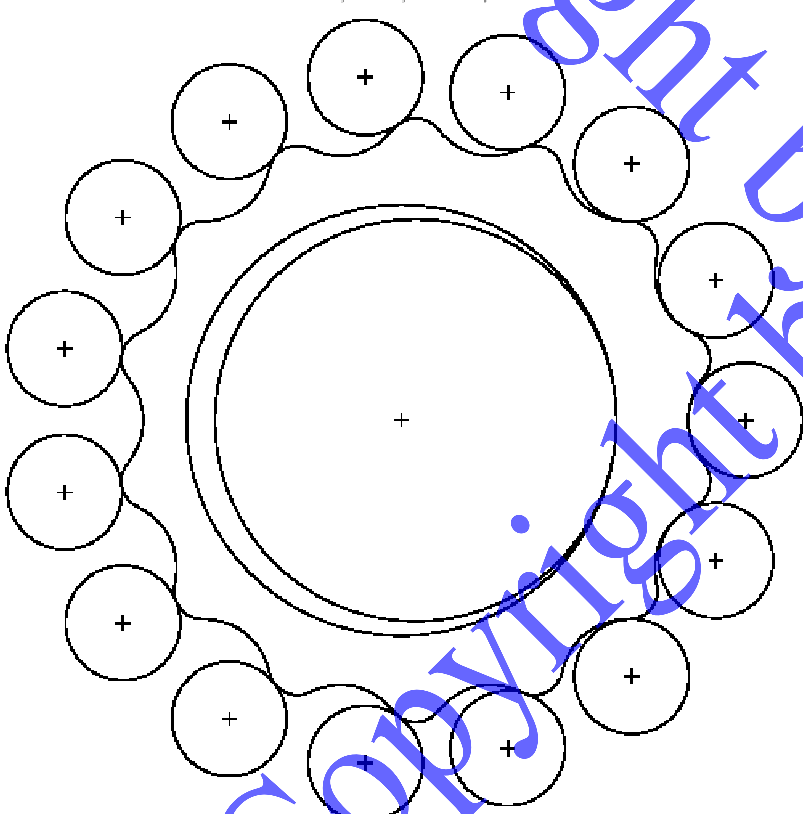
Anti-Backlash Transmission Design

- The principle of differential motion
- Commercial differential drives
  - Cycloidal drive
  - Harmonic drive
  - Epicyclic drive

Anti-Backlash Transmission Design

❐ Commercial Differential Drives

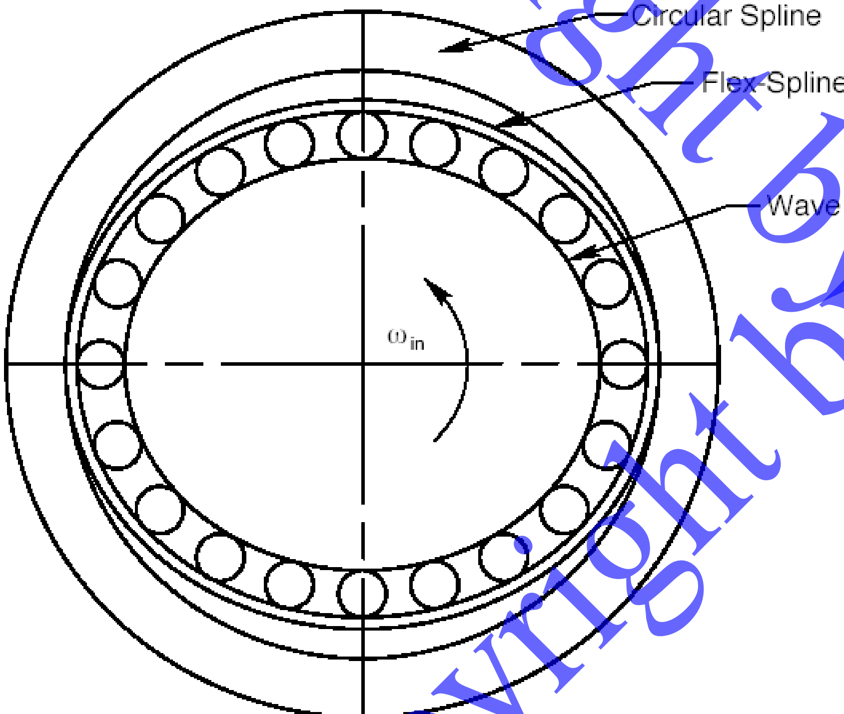
The Cycloidal Drive



The point where the two pitch circles contact is an instantaneous center meaning that all points on the cam instantaneously rotate about this center. In addition, each surface in contact with a roller is moving tangentially about the instant center. The normal force at each contact acts through the instant center. That point on the cam is instantaneously constrained against translations; only rotation is allowed.

❒ Commercial Differential Drives

The Harmonic Drive

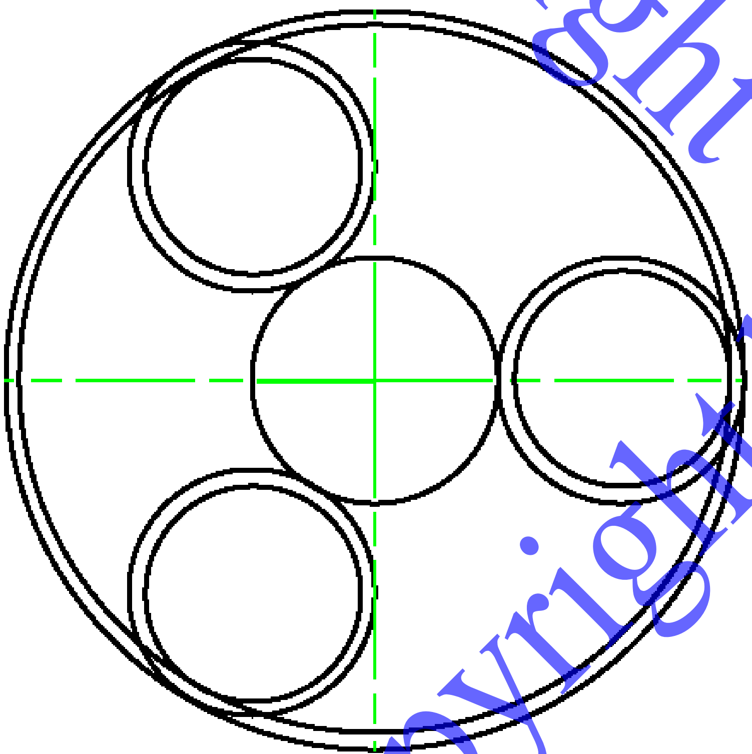


The three basic components of a harmonic drive are the circular spline, the flex-spline and the wave generator.

Anti-Backlash Transmission Design

❒ Commercial Differential Drives

The Epicyclic Drive



The circles represent the operating pitch diameters of the various gears. The planet gears have two operating pitch diameters because they mesh simultaneously with two different sized internal gears. One internal gear is fixed and the other is the output shaft. The sun gear is the input shaft.

In Real Life

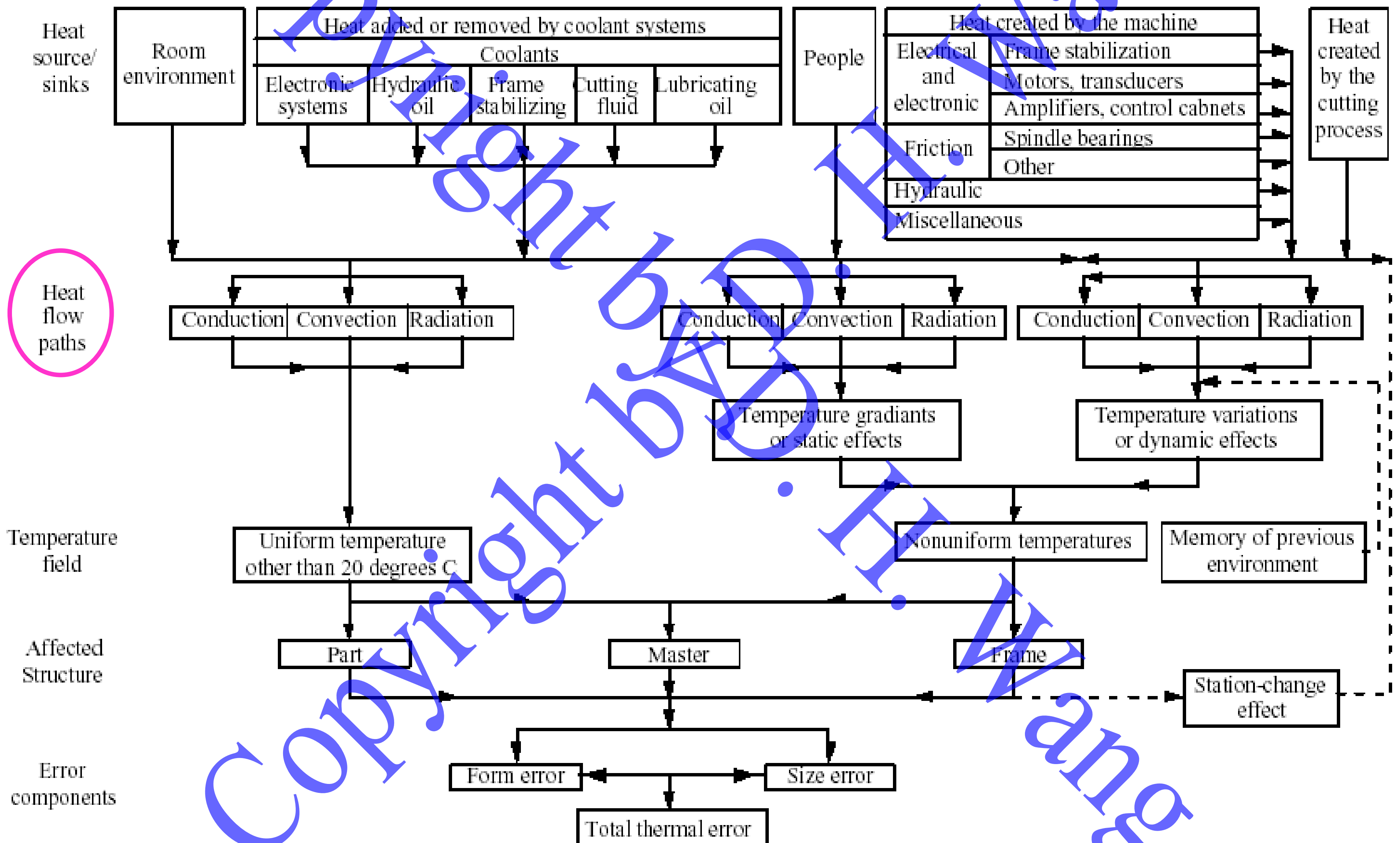
Thermal Growth Errors

- By far the most common cause of ***nonrepeatability*** is temperature.
  - More specifically, the cause is changing temperature with time, which causes thermal distortion of the structural loop (which consists of all the mechanical elements involved in holding the tool and work in a given relative position).
  - Interestingly, temperature problems also seem to be the least widely appreciated error source in machine tools. *[Donaldson, 1972]*

Thermal Growth Errors

- The international standard temperature, where a solid object has its true size, is 20° C (68° F).
  - This implies that the temperature distribution must be constant and uniform throughout the workpiece.
  - By international agreement, metrology at any other temperature contains thermally induced errors, although the systematic components can be reduced through compensation.
  - The thermally induced errors in the measuring machine or machine tool are usually more significant because of its size and complexity.

Heat sources and paths (After Bryan)



Thermal Growth Errors: Linear Expansion

- Simple to estimate
  - Axial expansion of tools, spindles and columns, caused by bulk temperature change ΔT , is often a significant error
  - At least it does not contribute to Abbe errors

$$\delta = \alpha L \Delta T$$

- Axial expansion in a gradient (one end stays at temperature, while the other end changes)

$$\delta = \frac{\alpha L\left(T_1 - T_2\right)}{2}$$

Thermal Growth Errors: Linear Expansion

- For a meter tall cast iron structure in a 1 Co/m gradient, δ = 5.5 um
  - This is a very conservative estimate, because the column will diffuse the heat to lessen the gradient

Thermal Growth Errors: Thermal Gradient

- One of the most common and insidious thermal errors
  - **Beam length = L, height = h, section I, gradient ΔT, straightness error**

$$\varepsilon_T = \frac{y}{\rho} = \frac{\alpha y \Delta T}{h} \qquad M = \frac{EI}{\rho}$$

$$\delta_T = \frac{M\left(L/2\right)^2}{2EI} = \frac{L^2\alpha\Delta T}{2h}$$

  - **Slope error at the ends of the beam (a=M(l/2)/EI)**

$$\theta_T = \frac{\alpha \Delta T L}{2h}$$

Thermal Growth Errors: Thermal Gradient

- One of the most common and insidious thermal errors
  - For a 1x1x0.3 m cast iron surface plate with DT=1/3 Co (1 Co/m), d = 1.5 mm and θT = 6.1 mrad.
    - This is a very conservative estimate, because the plate will diffuse the heat to lessen the gradient
  - In a machine tool with coolant on the bed, thermal warping errors can be significant
    - Angular errors are amplified by the height of components attached to the bed

Thermal Growth Errors: Thermal Gradient

- **Causes of thermal gradients**
  - The bed may be subjected to a flood of temperature controlled fluid
  - Evaporative cooling (common on large grinders)
  - Room temperature may vary wildly during the day
  - Overhead lights can create gradients in sensitive structures
    - Plastic PVC curtains are extremely effective at reducing infrared heat transmission
  - ...

Thermal Growth Errors: Thermal Gradient

- **Causes of thermal gradients**
  - **...**
  - **A large machine on a deep foundation (relies on the concrete for support), can have problems:**
    - **Several meters under the ground, the concrete is at constant temperature**
    - **The top of the machine and the concrete are at room temperature**
  - **Internal heat sources (motors, spindles, ballscrews, process)**

Thermal Growth Errors: Design Strategies

- Very troublesome because
  - they are always changing
  - components' heat transfer coefficients vary from machine to machine
- *Design strategies to minimize effects*
  - *Reduce sensitivity*
  - *Manage heat sources*
  - *Control machine environment*
  - *Compensate for measured deviations*

Thermal Growth Errors: Design Strategies

- **_Reduce sensitivity_**
  - ***Structural design***
    - **Obtain symmetric temperature distributions in symmetric structures to reduce distortions.**
  - ***Low-expansion materials***
    - **Use low CTE materials to reduce variations in geometry due to variations in temperature.**

Thermal Growth Errors: Design Strategies

- **_Manage heat sources (1/2)_**
  - **_Eliminate_**
    - **Place necessary heat sources outside the controlled environment and eliminate unnecessary heat sources.**
  - **_Reduce_: Use components that dissipate less heat.**
    - **_Maximize conductivity_**
  - **_Isolate_**
    - **Capture the heat near the source or prevent it from spreading into the structures.**
  - **Avoid cascading**
    - **……**
  - **Keep heat sources constant**

Thermal Growth Errors: Design Strategies

- ***Manage heat sources (2/2)***
  - ***Eliminate***
  - ***Reduce***
  - ***Isolate***
  - **Avoid cascading**
    - **Fluids that remove heat from an isolated source should return directly to the temperature control system rather than flowing over other sensitive parts of the system.**
  - **Keep heat sources constant**
    - **Necessary heat sources within the controlled environment such as lights should remain constant.**

Thermal Growth Errors: Design Strategies

- Control machine environment (1/2)
  - Control room air temperature
    - Reduce temperature variations in the machine by controlling the air temperature around it.
  - Isolate machine room
    - Prevent heat leakage into or out of the machine room to reduce variations in the room air temperature.
  - Isolate machine structure
    - Control the temperature of the metrology loop.
  - Control workpiece temperature
    - ……
  - Isolate machine operator

Thermal Growth Errors: Design Strategies

- Control machine environment (2/2)
  - Control room air temperature
  - Isolate machine room
  - Isolate machine structure
  - Control workpiece temperature
    - Use a temperature controlled fluid flowing over the workpiece to control its temperature. Consider the effect of viscous heating in high-speed fluid flows.
  - Isolate machine operator
    - The human body represents a heat source of about 100 watts. Precision applications may require the use of insulating clothing such as gloves.

Thermal Growth Errors: Design Strategies

- Compensate for measured deviations
  - Spindle growth compensation
    - Use a suitable displacement sensor such as a capacitance transducer to measure the axial spindle growth and compensate the z-axis position.
  - Metrology frame temperature
    - Measure the temperature of the metrology loop in sufficient detail to compute the error and compensate the slide positions.

Thermal Growth Errors: Design Strategies

- Conduction
  - Use thermal breaks (insulators)
  - Keep the temperature the same in the building all year!
  - Channel heat-carrying fluids (coolant coming off the process) away
- Convection
  - Use sheet metal or plastic cowlings
- Radiation
  - Plastic PVC curtains (used in supermarkets too!) are very effective at blocking infrared radiation
  - Use indirect lighting outside the curtains, & never turn the lights off!

Thermal Growth Errors: Design Strategies

- ❒ **Always ask yourself if symmetry can be used to minimize problems**

Summary of Lectures 2 and 3

- Principle Errors
  - How to determine the principle errors in the design stage and evaluating stage?
  - How to simplify the mechanism to assure the instrument's precision through compensating for the principle error?
- Manufacturing Errors
- Running Errors
  - Force/Stress induced deflection
  - Measuring force induced deflection
  - Wear
  - Clearance and backlash induced error
  - Thermal induced error
  - Vibration
  - Disturbance and environment condition

The End

- ***Thank you very much for your attention!***

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