Lecture 2: Fundamental Principles

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Question: Am I a teacher as that in the cartoon?

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Outline (1/3)

- Intro. to Fundamental Principles
- Fundamental Principles
  - Occam's Razor: KISS & MISS
  - Abbe's Principle and its Extension
  - Min {Deformation}
  - Comparative Measurement Principle
  - Reading Averaging
  - Shortest Measuring Chain/Minimize the Metrology Loop
  - Unify the Coordinate Systems
  - Next Slide…

Focus

Outline (2/3)

- **Fundamental Principles**
  - Precision Matching Principle
  - Economy Principle
  - Saint-Venant's Principle
  - Exact Constraint Design
  - Elastic Averaging
  - Centers-of-Action
  - The Golden Rectangle
  - Compensating Principle
  - Separation of Metrology and Structural Loops
  - Error Separation Techniques

Outline (3/3)

- Summary
- Acknowledgement

Intro. to Fundamental Principles

- 设计原理与设计原则
  - 在仪器设计长期实践的基础上，设计者经过不断的总结经验、继承和发展前人的科技成果，形成的一些带有普遍性的仪器设计所应遵循的基本原则与基本原理
- 本章介绍的设计原理与原则
  - Occam’s Razor: KISS & MISS
  - Abbe’s Principle and its Extension
  - Min {Deformation}
  - Comparative Measurement Principle
  - Reading Averaging
  - **Next slide…**

Intro. to Fundamental Principles

❒ 本章介绍的设计原理与原则

- Shortest Measuring Chain/Minimize the Metrology Loop
- Unify the Coordinate Systems
- Precision Matching Principle
- Economy Principle
- Saint-Venant's Principle
- Exact Constraint Design
- Elastic Averaging
- Centers-of-Action
- Next slide…

Intro. to Fundamental Principles

❒ 本章介绍的设计原理与原则

- The Golden Rectangle
- Compensating Principle
- Separation of Metrology and Structural Loops
- Error Separation Techniques

Occam's Razor: KISS & MISS

William of Ockham, born in the village of Ockham in Surrey (England) about 1285

- William of Occam (or Ockham) (1284-1347) was an English philosopher and theologian
  - Ockham stressed the Aristotelian principle that entities must not be multiplied beyond what is necessary

http://wotug.ukc.ac.uk/parallel/www/occam/occam-bio.html

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Occam's Razor: KISS & MISS

- William of Occam (or Ockham) (1284-1347) was an English philosopher and theologian
  - "Ockham wrote fervently against the papacy in a series of treatises on papal power and civil sovereignty. The medieval rule of parsimony, or principle of economy, frequently used by Ockham came to be known as Ockham's Razor. The rule, which said that *plurality should not be assumed without necessity* (or, in modern English, ***keep it simple, stupid***), was used to eliminate many pseudo-explanatory entities"

Occam's Razor: KISS & MISS

❒ **William of Occam (or Ockham) (1284-1347) was an English philosopher and theologian**

- **A problem should be stated in its basic and simplest terms**
- **The simplest theory that fits the facts of a problem is the one that should be selected**
- **Limit analysis is an invaluable way to identify and check simplicity**

Occam's Razor: KISS & MISS

- Use fundamental principles as catalysts to help you
  - **Keep It Super Simple (*KISS*)**
  - **Make It Super Simple (*MISS*)**
  - **Because *"Silicon is cheaper than cast iron…"* (Don Blomquist)**

Be aware of complexity but make it simple!

The End

Thank you very much for your attention!

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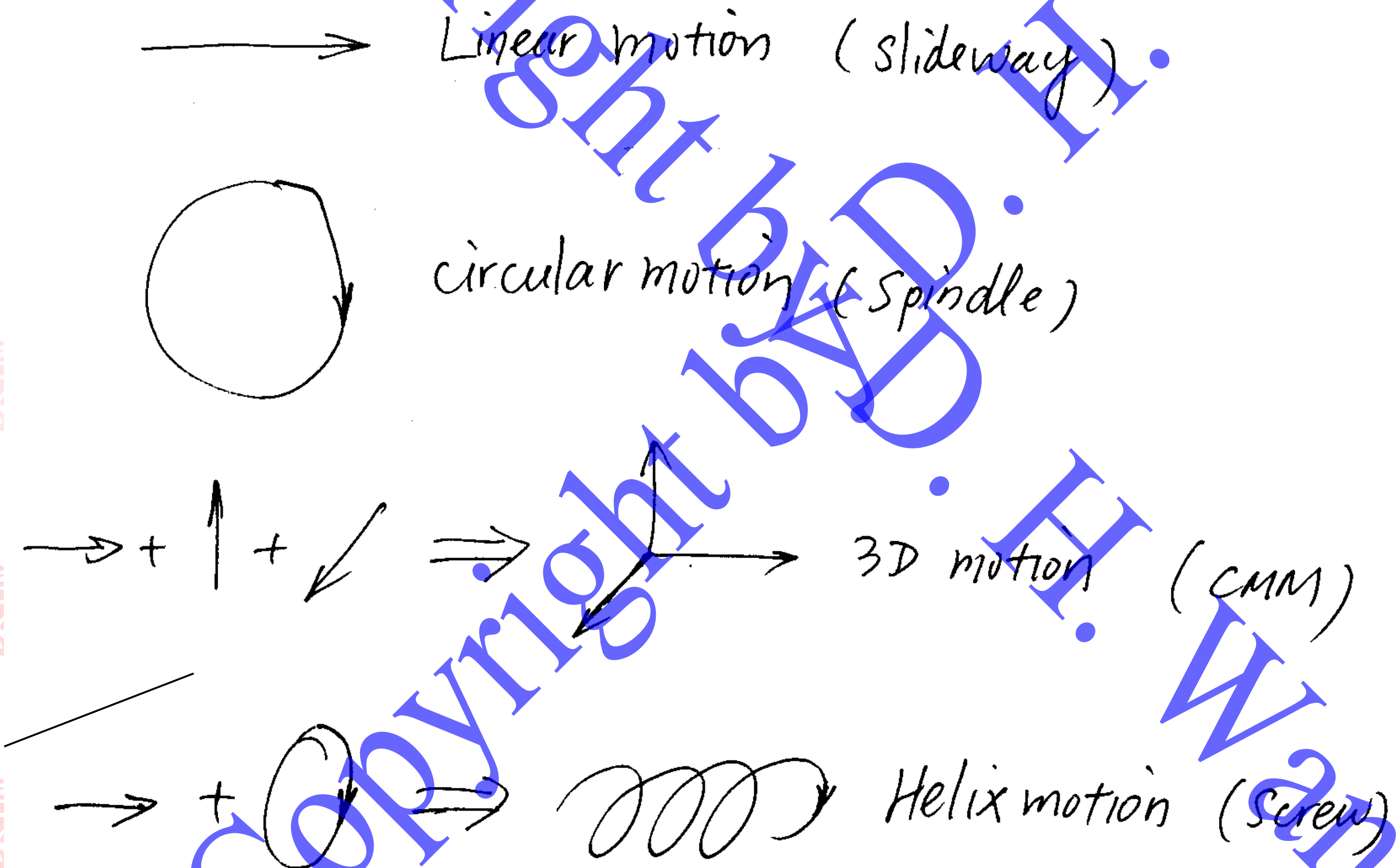
Comparative Measurement Principle: Outline

- Introduction
- Displacement-Synchronized Comparative Measurement Principle (DSCMP)
- Differential Comparative Measurement Principle
- Zero-State Comparative Measurement Principle

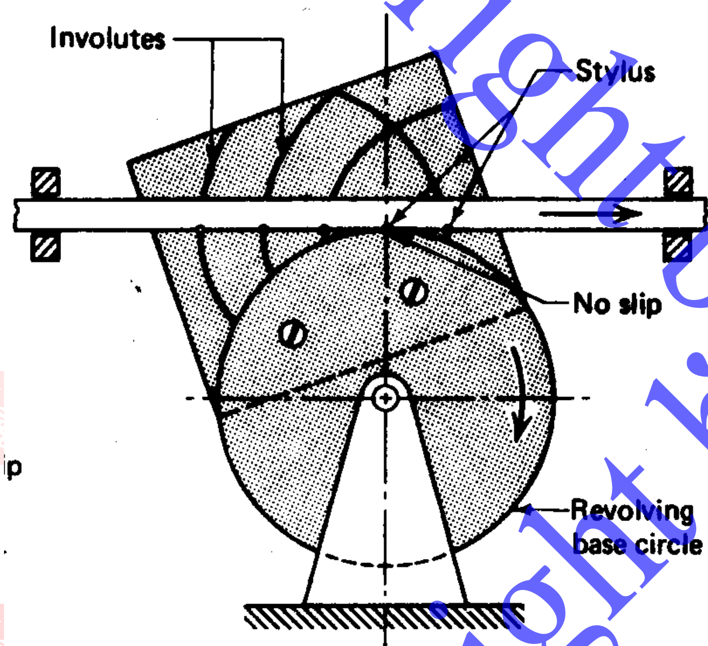
Introduction

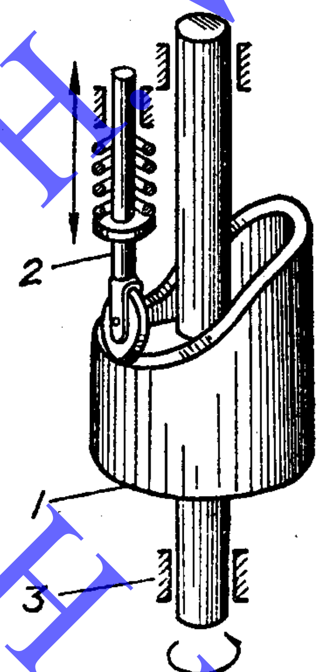
- ❒ **Basic Concept**
- ❒ **Types**
  - **Displacement-synchronized comparative measurement principle (DSCMP)**
  - **Differential comparative measurement principle**
  - **Zero-state comparative measurement principle**

❒ Motion and its combination



❒ Motion and its seperation





Revolving base circle, the beam and base circle roll with each other without slip

DSCMP

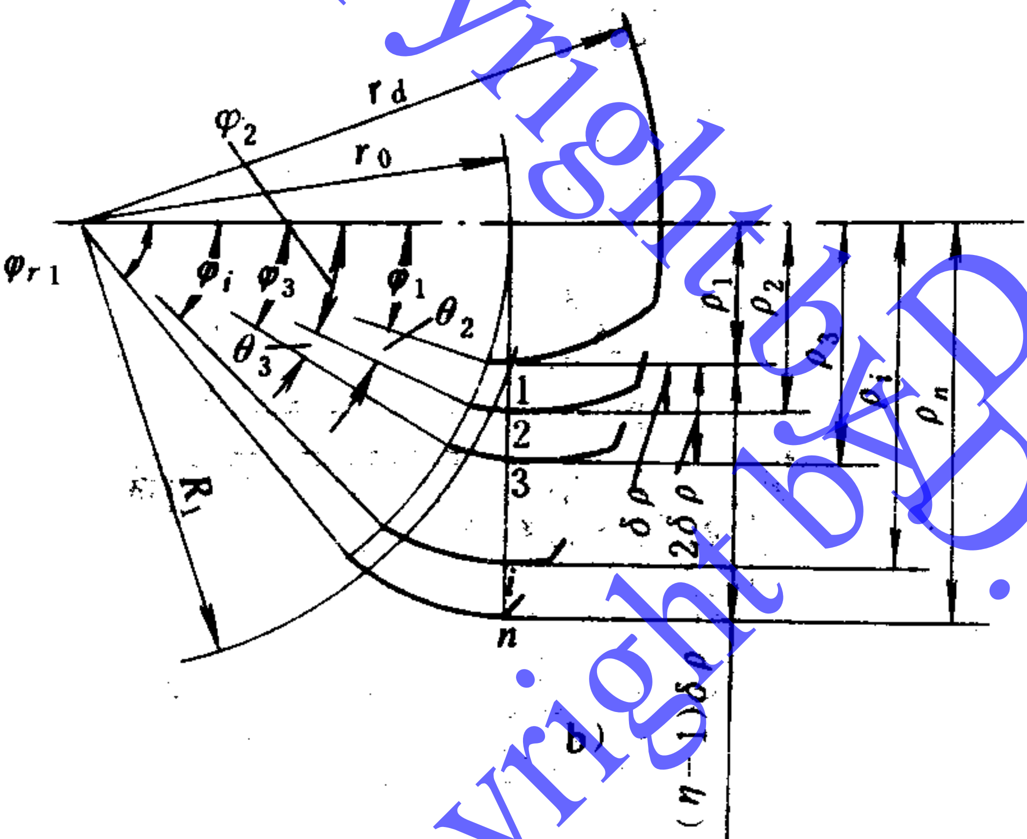
❒ **Definition**

- 对复合参数进行测量的近代方法是先分别用激光装置或光栅装置等测出它们各自的位移量，然后再根据它们之间存在的特定关系由计算机系统直接进行运算比较而实现测量

❒ **Ex:** ***Universal Whole Tooth Error Measuring Machine***

Universal Whole Tooth Error Measuring Machine

- ❒ **Theoretical**

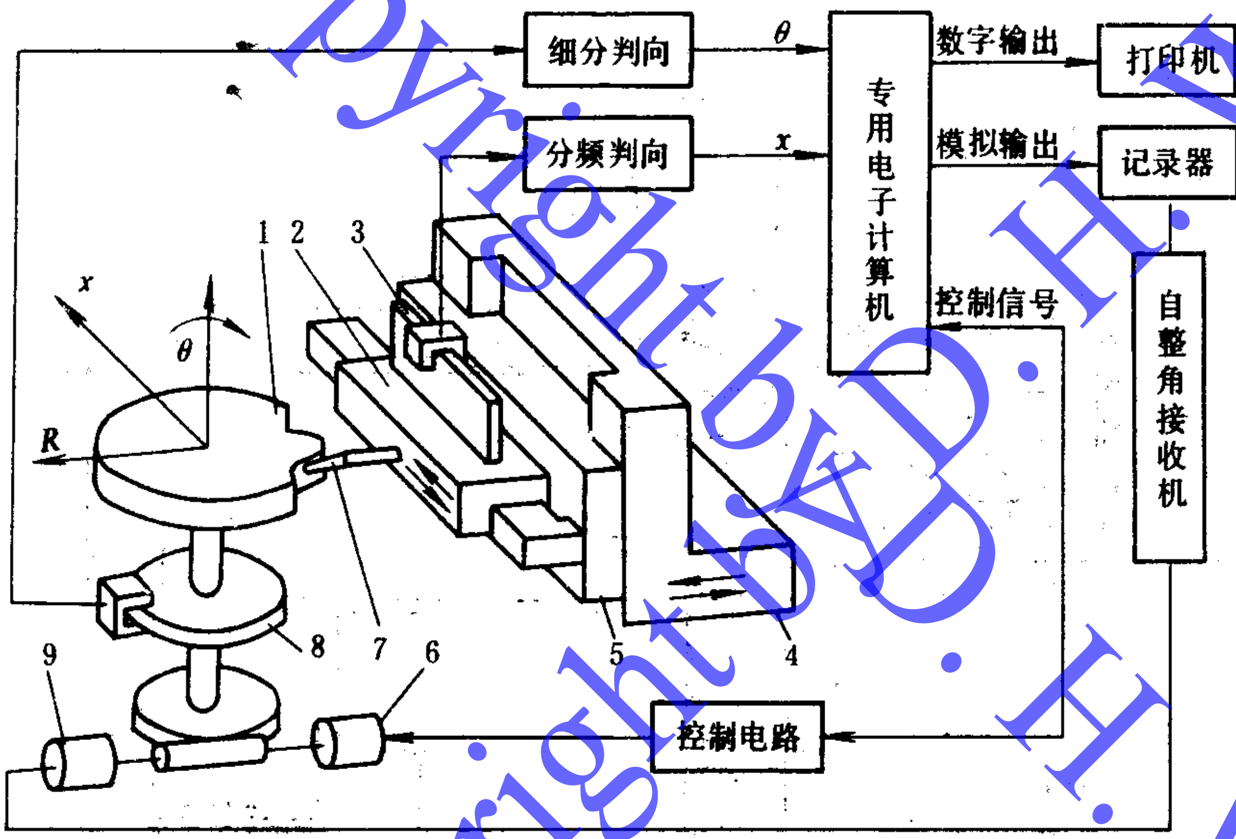


$$\rho_i = r_0\varphi_i = r_0(\theta_i + \varphi_{i-1})$$

$$\rho_i' = r_0\varphi_i' = r_0(\theta_i' + \varphi_{i-1}')$$

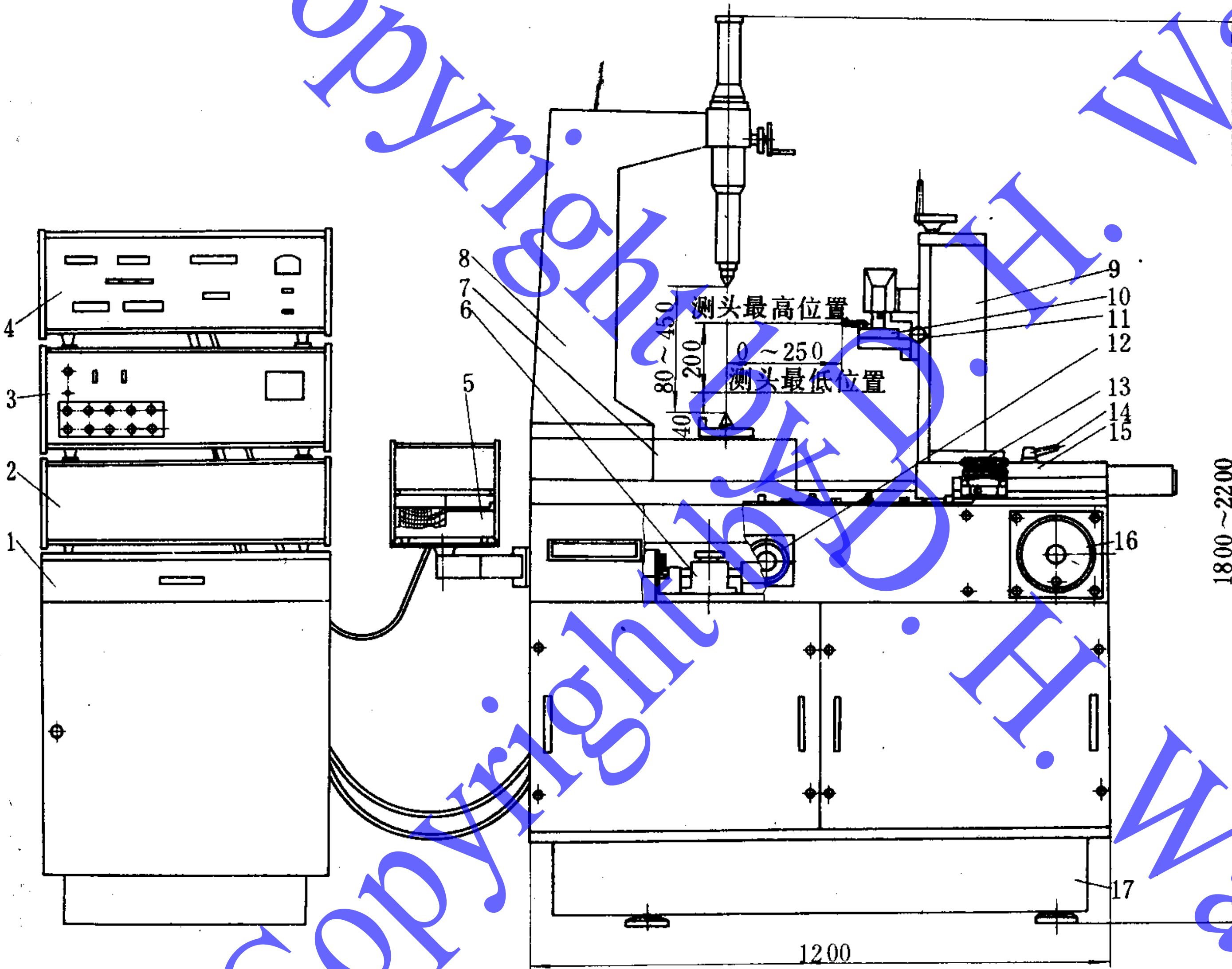
Universal Whole Tooth Error Measuring Machine

❒ Principle



1-Gear; 2-Tangent slide way; 3-Length grating; 4-Radial slide way; 5-Vertical slide way; 6-Servo motor; 7-Pick up; 8-Round grating.

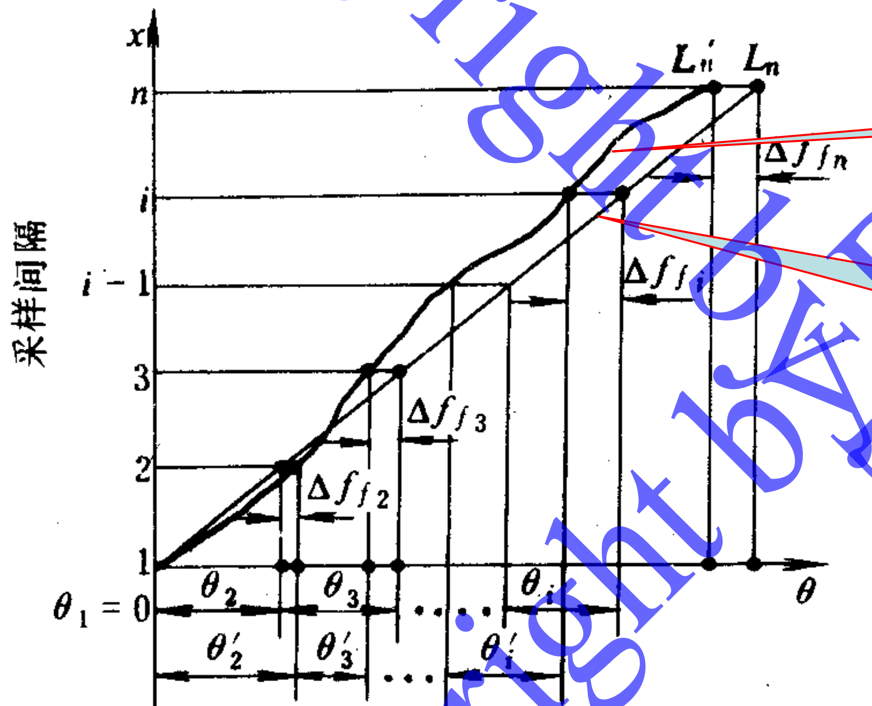
Universal Whole Tooth Error Measuring Machine



- 3001 Universal Whole Tooth Error Measuring Machine

Universal Whole Tooth Error Measuring Machine

❒ **Mathematical model of the profile error**



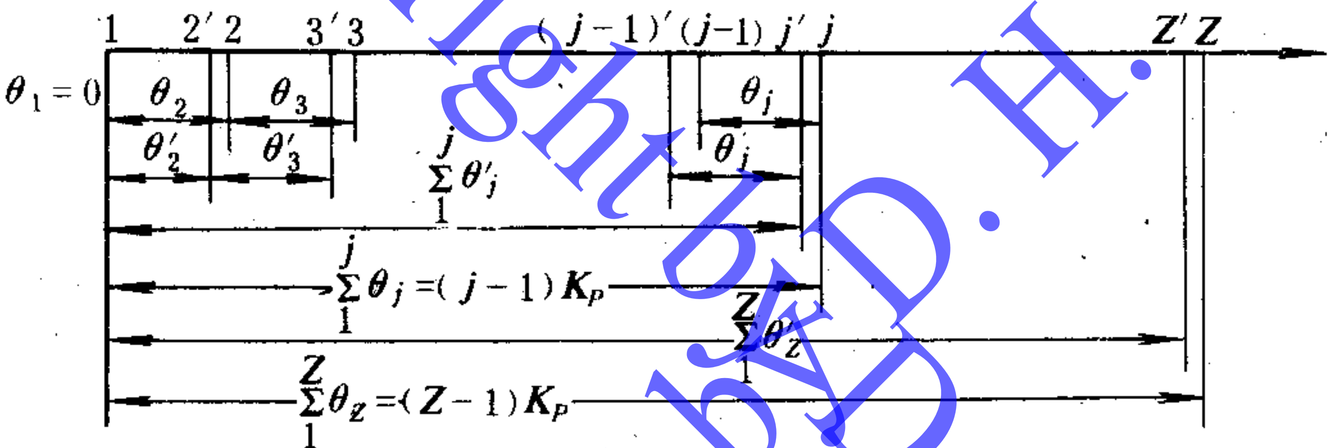
Practical Involute Profile

Theoretical Involute Profile

$$K_f = \frac{1296000\delta_\rho}{2\pi r_b a}$$

$$\Delta f_{fi} = \sum_{i=1}^{n}\theta'_i - \sum_{i=1}^{n}\theta_i = \sum_{i=1}^{n}\theta'_i - (i-1)K_f$$

❒ Mathematical model of the pitch error

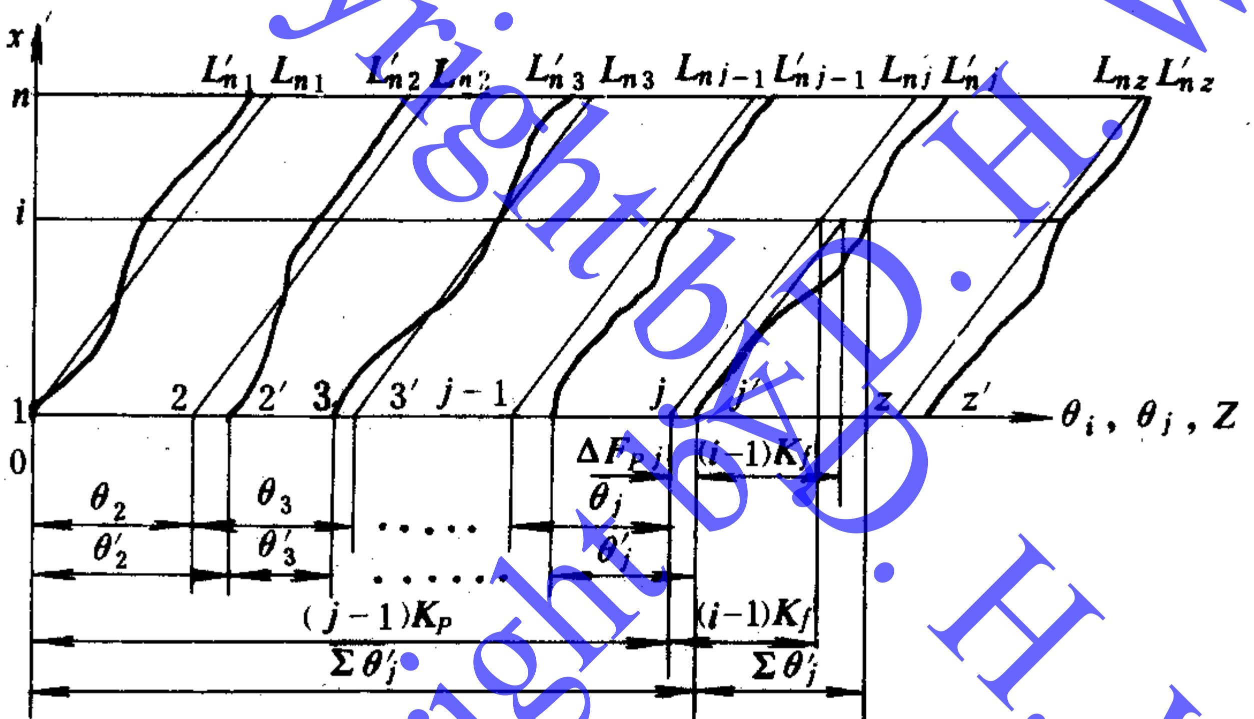


$$\Delta F_{pj} = \sum_{j=1}^{z} \theta'_j - (j-1) K_p$$

$$K_p = \frac{1296000}{za}$$

Universal Whole Tooth Error Measuring Machine

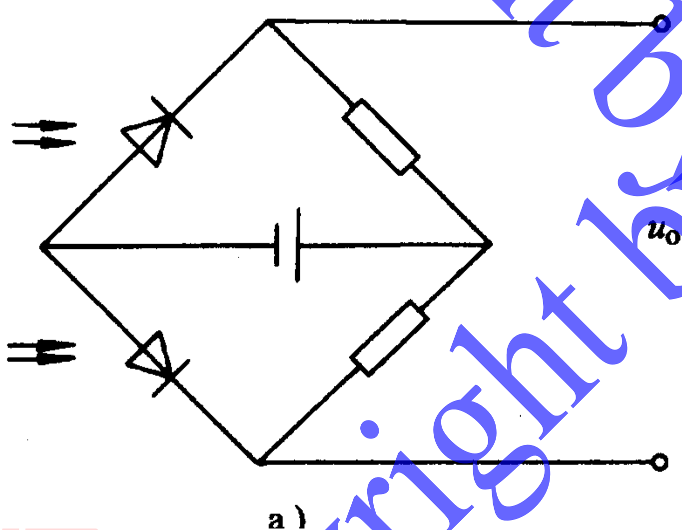
❒ **Mathematical model of the whole tooth error**

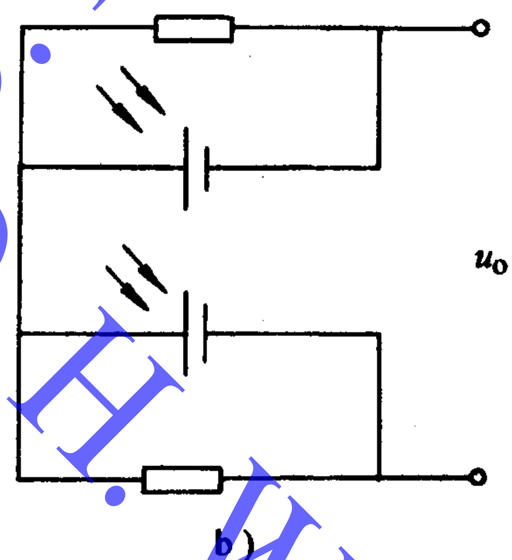


$$\Delta\theta_{ij} = \sum_{j=1}^{z}\sum_{i=1}^{n}\theta'_{ij} - \left[\left(j-1\right)K_p + \left(i-1\right)K_f\right]$$

Differential Comparative Measurement Principle

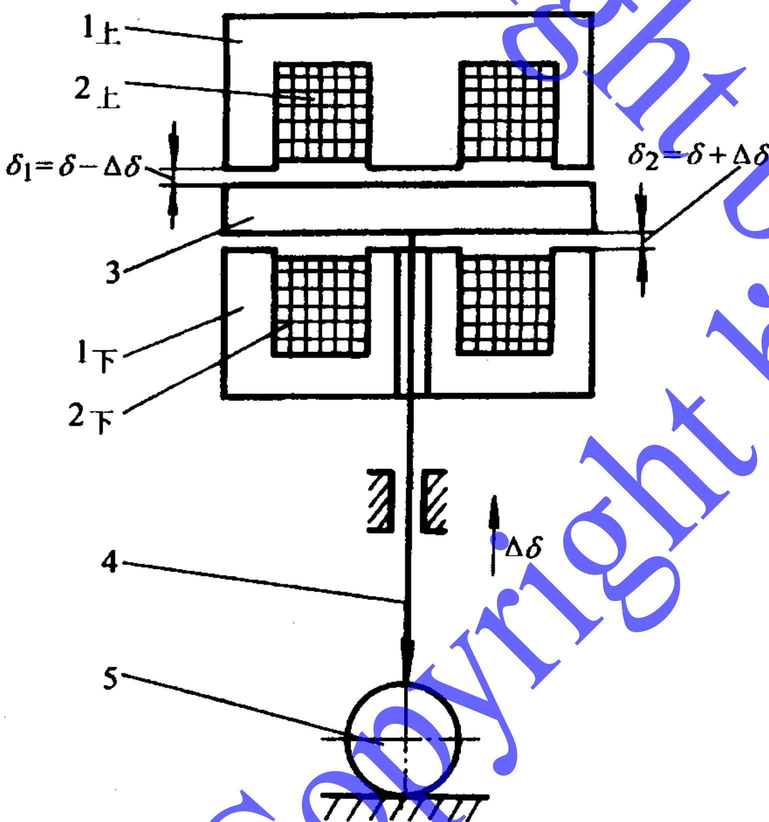
- Differential comparative measurement principle for electronic quantities

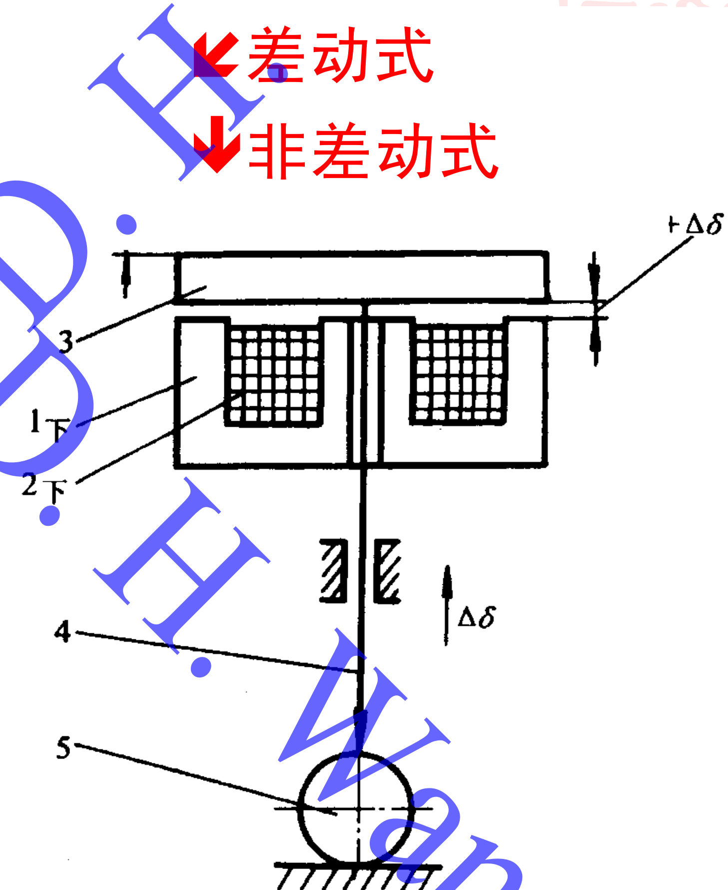




Differential Comparative Measurement Principle

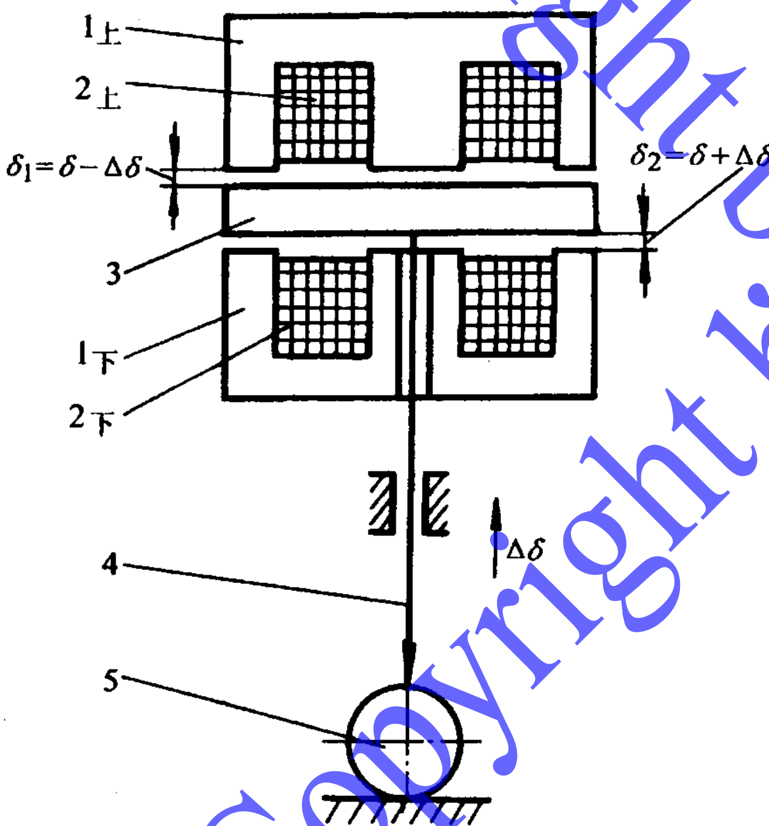
- Differential comparative measurement principle for electronic quantities





Differential Comparative Measurement Principle

- Differential comparative measurement principle for electronic quantities



←差动式

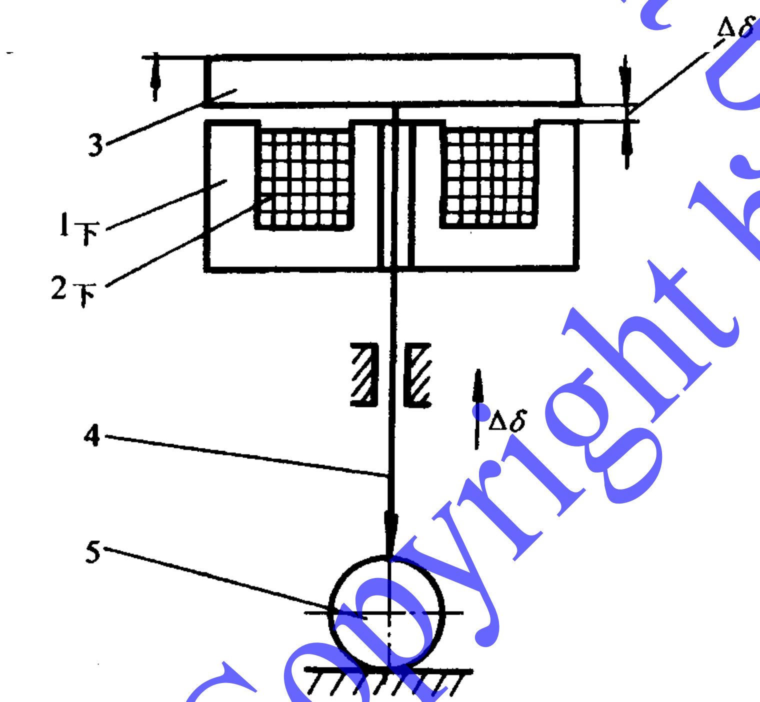
$$\Delta L = (L+\Delta L_1) - (L-\Delta L_2) = \frac{w^2\mu_0 S}{2(\delta-\Delta\delta)} - \frac{w^2\mu_0 S}{2(\delta+\Delta\delta)} \approx 2\left(L\frac{\Delta\delta}{\delta}\right)$$

↑非差动式

$$\Delta L = \frac{w^2\mu_0 S}{2(\delta-\Delta\delta)} - \frac{w^2\mu_0 S}{\delta} \approx L\left(\frac{\Delta\delta}{\delta}\right)$$

Differential Comparative Measurement Principle

❒ **Differential comparative measurement principle for electronic quantities**



←差动式

$$\Delta L = (L + \Delta L_1) - (L - \Delta L_2)$$

$$= \frac{w^2 \mu_0 S}{2(\delta - \Delta\delta)} - \frac{w^2 \mu_0 S}{2(\delta + \Delta\delta)}$$

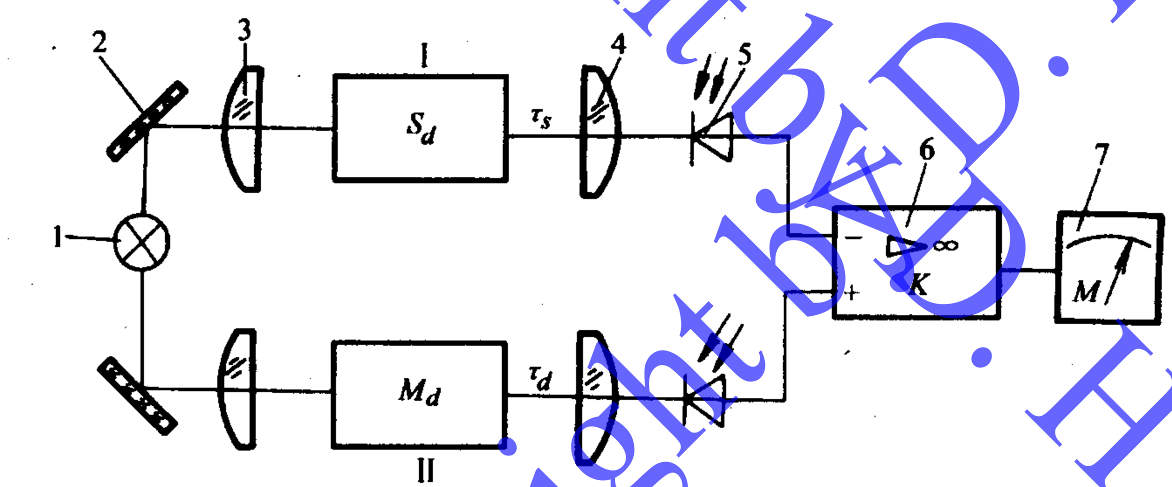
$$\approx 2\left(L \frac{\Delta\delta}{\delta}\right)$$

↑非差动式

$$\Delta L = \frac{w^2 \mu_0 S}{2(\delta - \Delta\delta)} - \frac{w^2 \mu_0 S}{\delta} \approx L\left(\frac{\Delta\delta}{\delta}\right)$$

Differential Comparative Measurement Principle

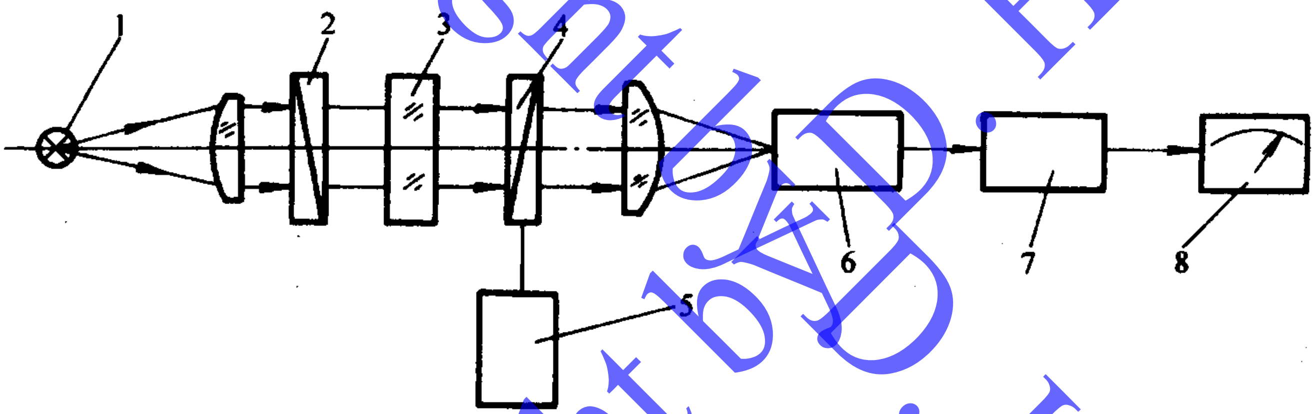
- **Differential comparative measurement principle for optical quantities**
  - 透过率测量系统



$$\theta = KMS_I \phi_0 (\tau_d - \tau_s)$$

Zero-State Comparative Measurement Principle

- Differential comparative measurement principle for optical quantities



$$\theta = KS_I \phi_0 M(\theta_x - \theta_0)$$

The End

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Shortest Metrological Loop

- 测量链与结构链
- 机械式VS电子创成式

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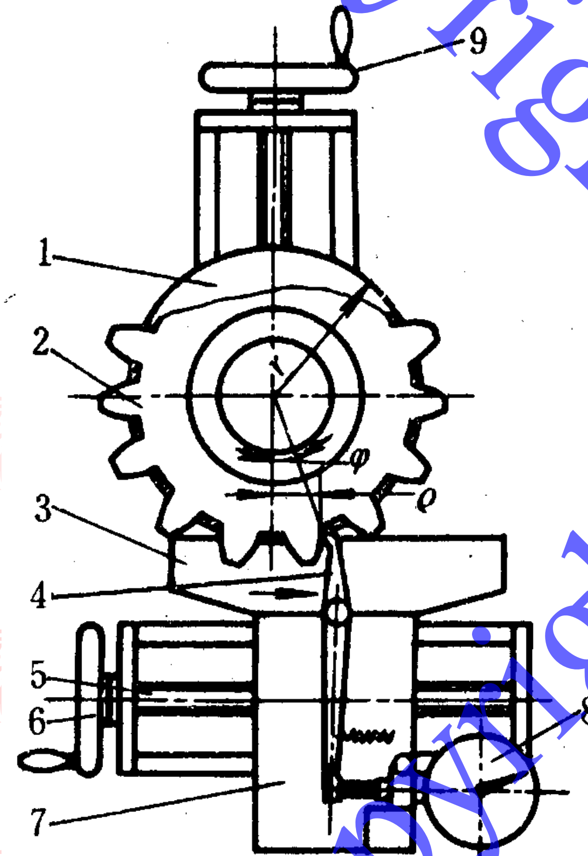
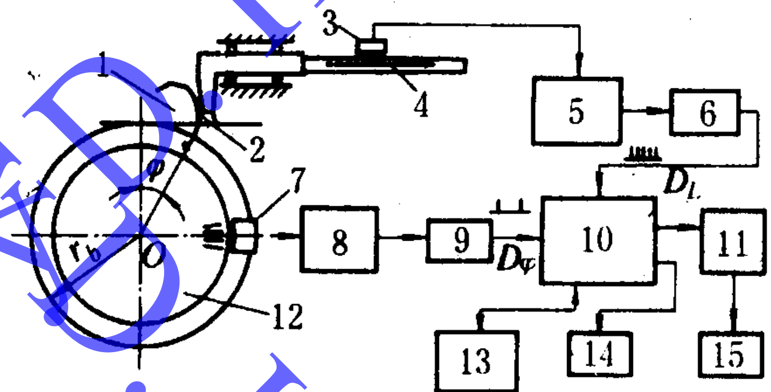
Shortest Metrological Loop

❒ 机械式VS电子创成式





Shortest Metrological Loop

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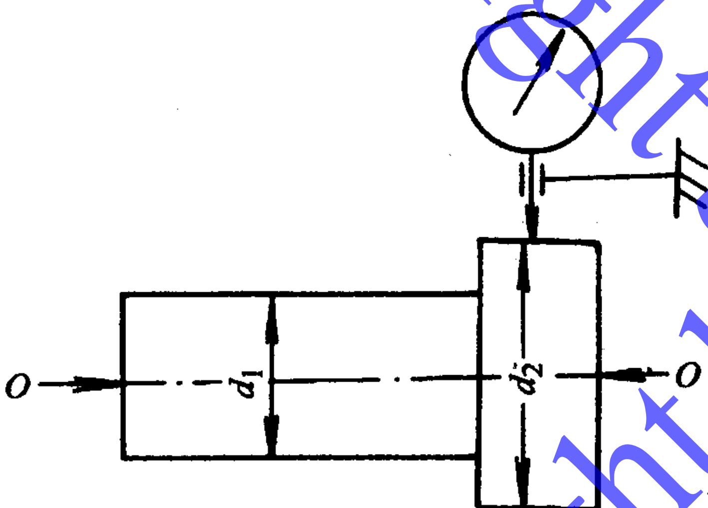
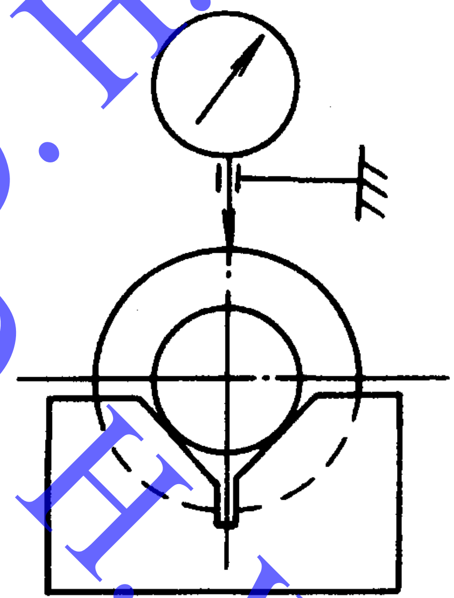
Unify Coordinate Systems: Outline

- ❒ 加工基准
- ❒ 测量基准
- ❒ 工作基准

Unify Coordinate Systems

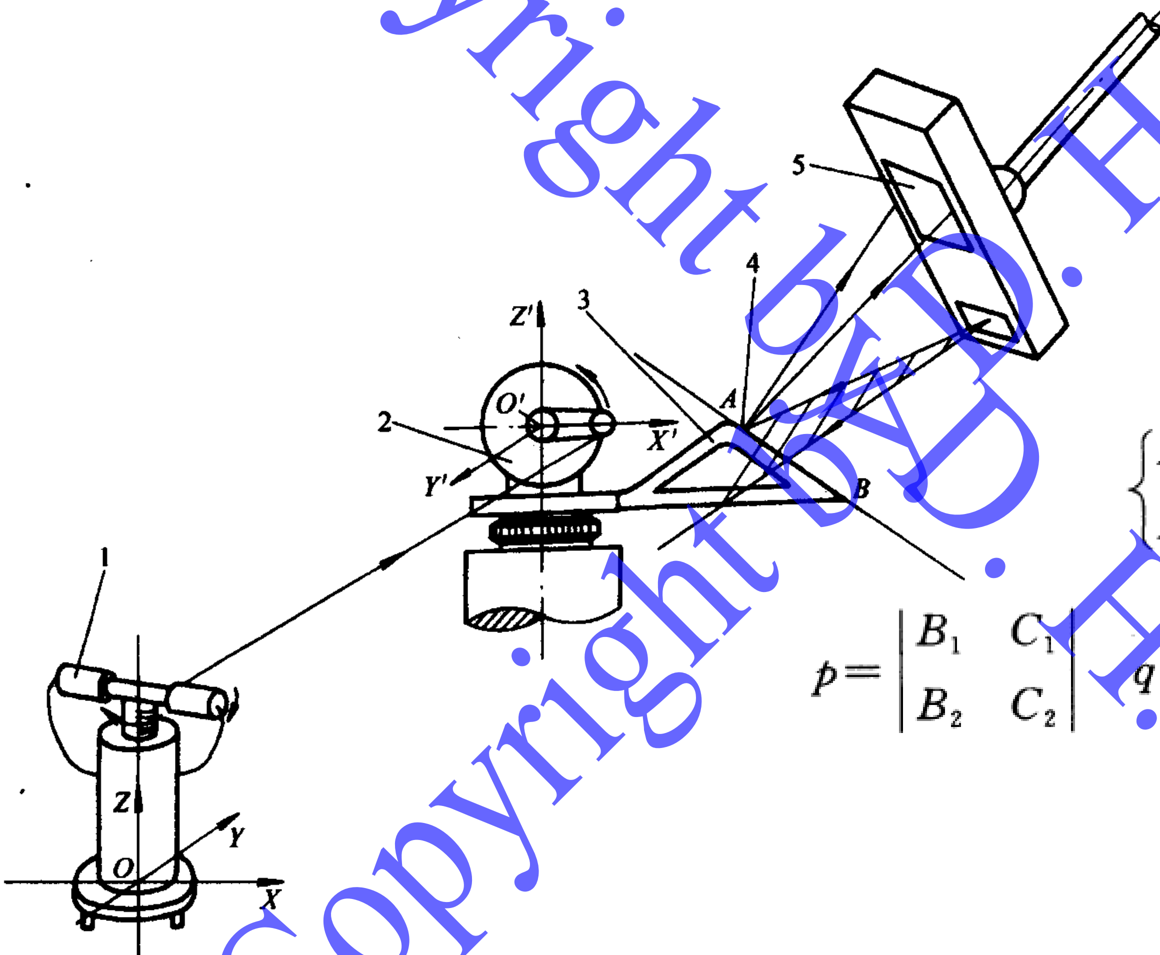
❐ 测量基准VS加工基准





Unify Coordinate Systems

❒ **Measuring Automobile Body-in-White**



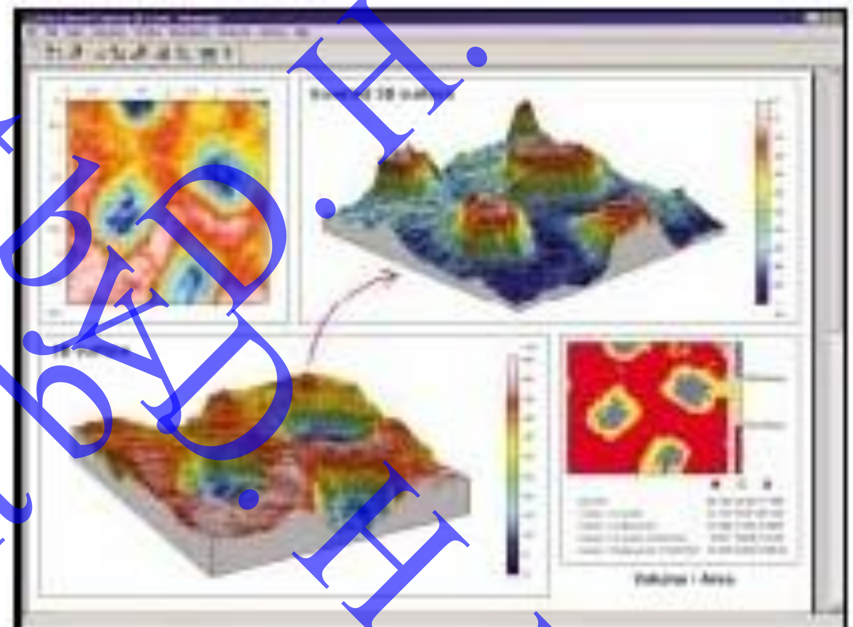
$$\begin{cases} A_1X' + B_1Y' + C_1Z' + D_1 = 0 \\ A_2X' + B_2Y' + C_2Z' + D_2 = 0 \end{cases}$$

$$p = \begin{vmatrix} B_1 & C_1 \\ B_2 & C_2 \end{vmatrix} \quad q = \begin{vmatrix} C_1 & A_1 \\ C_2 & A_2 \end{vmatrix} \quad r = \begin{vmatrix} A_1 & B_1 \\ A_2 & B_2 \end{vmatrix}$$

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = R_\theta \begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} + T$$

Precision Matching Principle

- 高精度表面轮廓测量



http://www.taylor-hobson.com

Precision Matching Principle

Vertical Performance	PGI Plus	PGI Standard
Nominal measuring range (Z)	10mm [60mm stylus arm] (0.39in [2.36in]) 20mm [120mm stylus arm] (0.79in [4.72in]) 25mm [150mm stylus arm] (0.98in [5.90in])	
Resolution (Z) [4]	0.8nm @10mm range (0.03µin @ 0.39in) range	12.8nm @10mm range (0.5µin @ 0.39in) range
Range to resolution ratio	12,480,000 : 1	780,000 : 1
Stylus arm length, tip size, force	60mm arm, 2µm radius conisphere diamond stylus, 1mN force 120mm arm, 0.5mm radius ball, 20mN force - optional	
Z axis non-linearity, (Z = gauge displacement)	(0.07 + 0.03 Z (mm)) µm (3 + 30 Z (inches)) µin - after calibration [5]	
Repeatability of Z axis indication	Flat surface - typically 0.05um (2µin) [6] – Curved surface - typically 0.10µm (4µin) [7]	

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Economy Principle

Economy Principle

- 工艺性
- 合理的精度要求
- 合理选材
- 合理的调整环节
- 提高仪器寿命
- 尽量使用标准件和标准化模块

Saint-Venant's Principle: Outline

Saint-Venant's Principle

http://www-gap.dcs.st-and.ac.uk/~history/Mathematicians/Saint-Venant.html

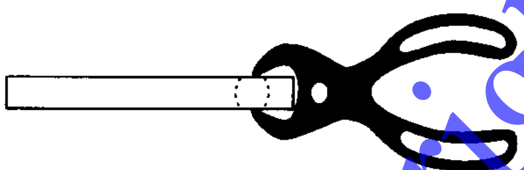
- Saint-Venant did extensive research in the theory of elasticity, and many times he relied on the assumption that ***local effects of loading do not affect global strains***
  - e.g., bending strains at the root of a cantilever are not influenced by the local deformations of a point load applied to the end of a cantilever

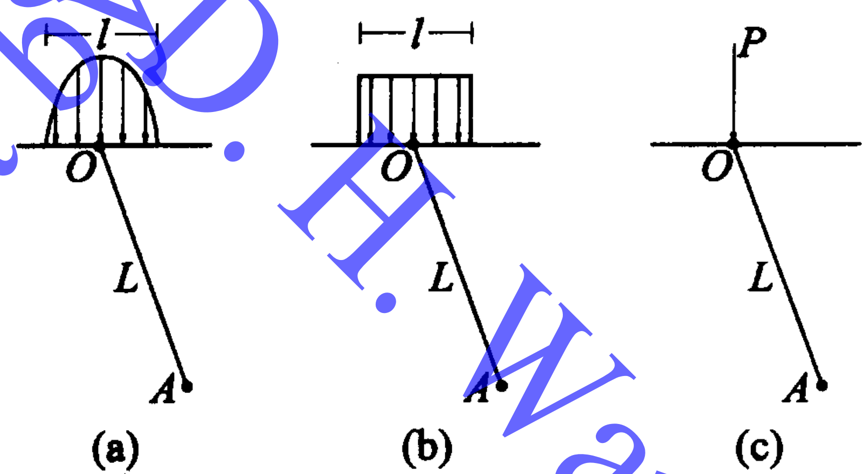
Saint-Venant's Principle

- The following was stated by Barré de Saint-Venant in *Mém. savants étrangers*, vol. 14, 1855.
  - **If the forces acting on a small portion of the surface of an elastic body are replaced by another statically equivalent system of forces acting on the same portion of the surface, this redistribution of loading produces substantial changes in the stresses locally but has a negligible effect on the stresses at distances which are large in comparison with the linear dimensions of the surface on which the forces are changed.**

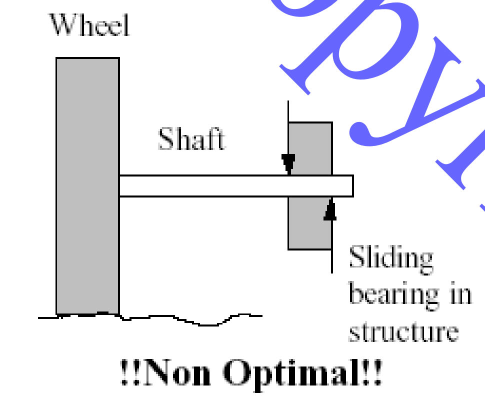
Saint-Venant's Principle

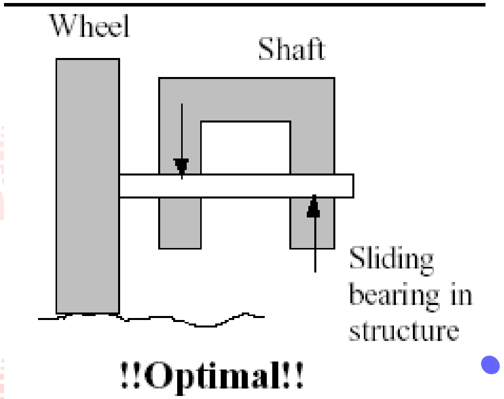
- The engineering applications of his general observations are profound for the development of conceptual ideas and initial layouts of designs:
  - To NOT be affected by local deformations of a force, be several characteristic dimensions away

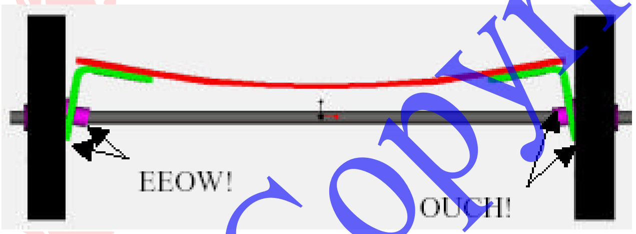




Saint-Venant's Principle



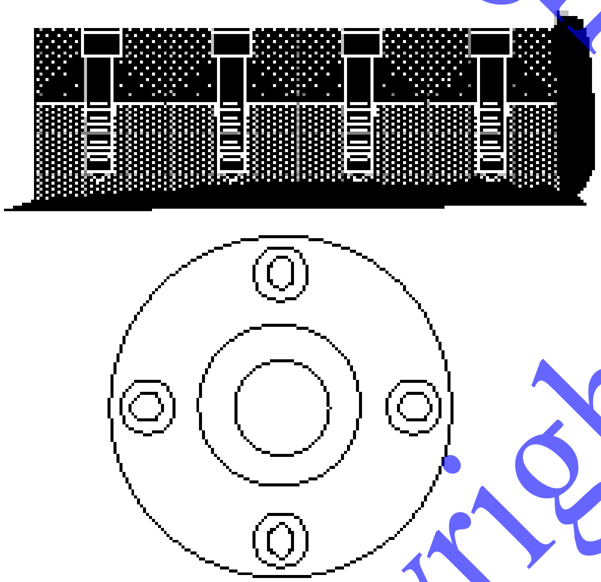




- Saint-Venant: *Linear Bearings*
  - L/D>1, 1.6:1 very good, 3:1 super ideal
- Saint-Venant: *Rotary Bearings*
  - L/D>3 if you are to have the bearings "build the shaft into a wall"
  - IF L/D<3, BE careful that slope from shaft bending does not KILL the bearing!

Saint-Venant’s Principle

- Bolting bearings in pace: beware the zone under a bolt that deforms due to bolt pressure!



Saint-Venant's Principle: Structures

- To NOT feel something's effects, be several characteristic dimensions away!
  - If a plate is 5 mm thick and a bolt passes through it, you should be 3 plate thicknesses away from the bolt force to not cause any warping of the plate!
  - Many bearing systems fail because bolts are too close to the bearings

Saint-Venant's Principle

- To dominate and control something, control several characteristic dimensions
- If a column is to be cantilevered, the anchor region should be 3 times the column base area
- Most machines that suffer from "lawn furniture syndrome" have inadequate anchoring

The End

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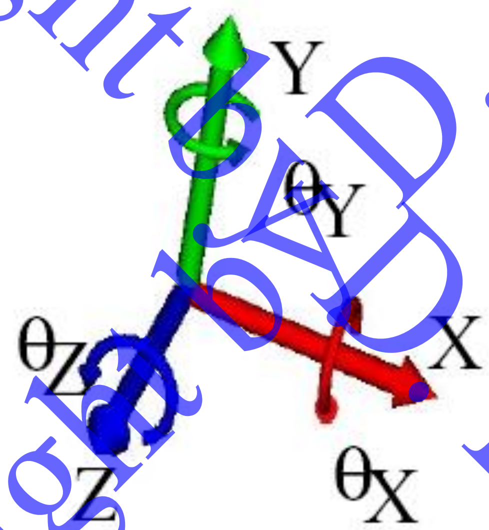
6-DOF

❒ **Every rigid body has 6 Degrees-of-Freedom (DOF)**

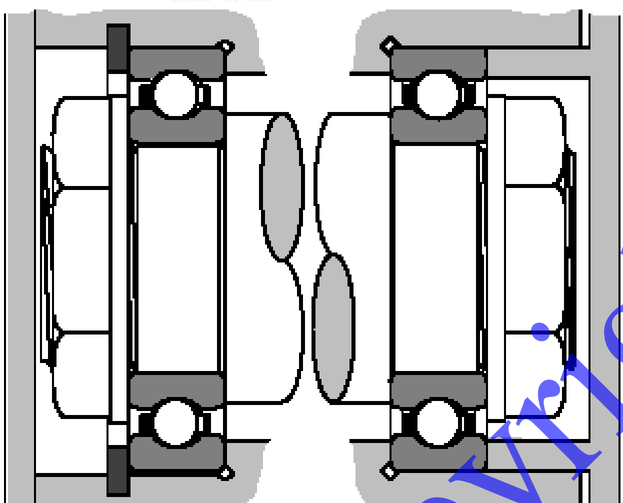


Exact Constraint Design

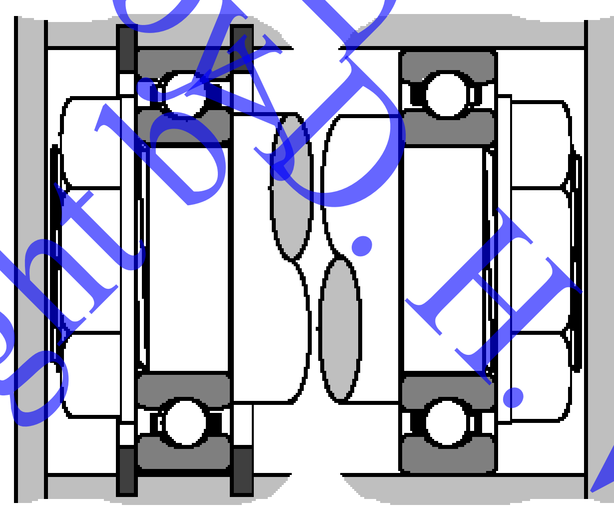
- ❒ **Make sure you have constrained what you want to constrain!**
  - **For a body to have N degrees of freedom free to move, there must be 6-N bearing reaction points!**
  - **To resist translation, a force is required.**
  - **To resist rotation, a moment, or two forces acting as a couple, is required!**
- ❒ **Saint-Venant rules! Do not constrain a shaft with more than 2 bearings, unless it is very long…**

Exact Constraint Design

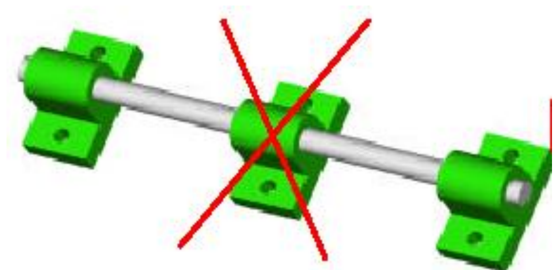
- Make sure you have constrained what you want to constrain!
- Saint-Venant rules! Do not constrain a shaft with more than 2 bearings, unless it is very long…



Overconstrained

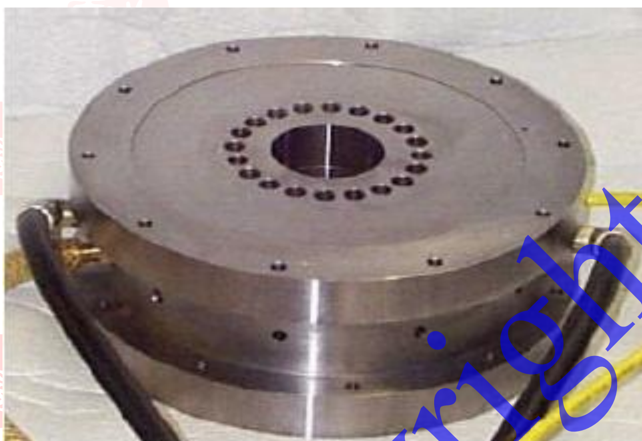


Properly constrained



Elastic Averaging

- *Elastic Averaging: Error Management*
- "Random results are the result of random procedures"
- How is something made from parts accurate to 5 microns, accurate as an assembly to 0.05 microns?



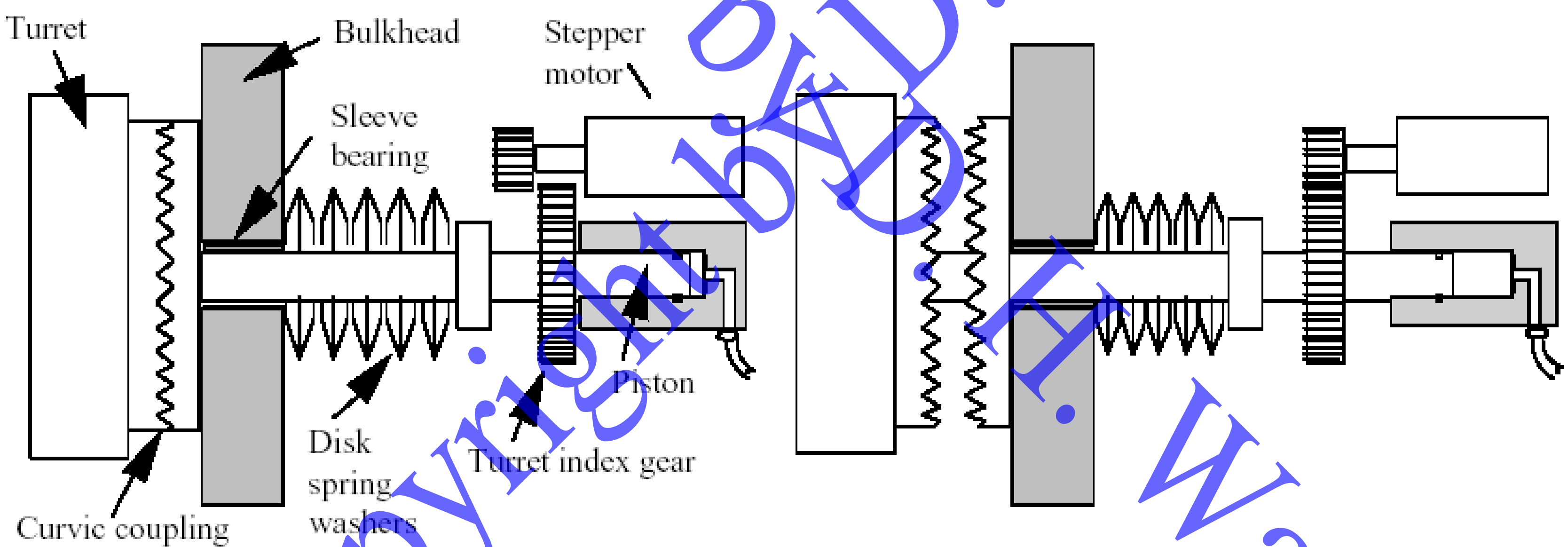
Rotary Table with 50 nanometer radial error motion designed by Prof. Slocum's graduate students Nathan Kane and Joachim Sihler

Elastic Averaging

- Any one error can be averaged out by having many similar features
  - As in gathering data with random errors, the accuracy of the reading is proportional to the square root of the number of samples taken
- The term *elastic averaging* describes a condition where two objects are connected through many points of contact in a highly over-constrained manner.

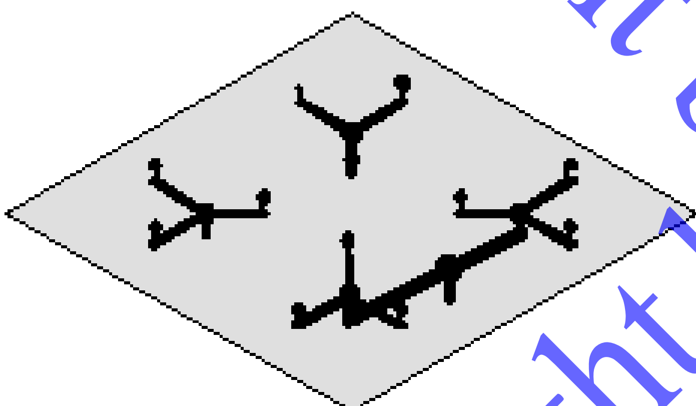
Ex.

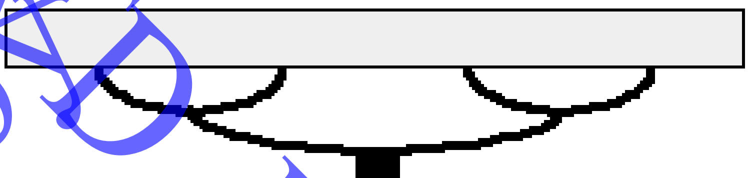
- Indexing tables often use a *curvic* coupling (two face gears meshed together) to achieve a high degree of elastic averaging (accuracy), stiffness, and load capacity



Ex.

- Windshield wiper blades and surface plate or large-mirror supports distribute the loads using *wiffle trees*



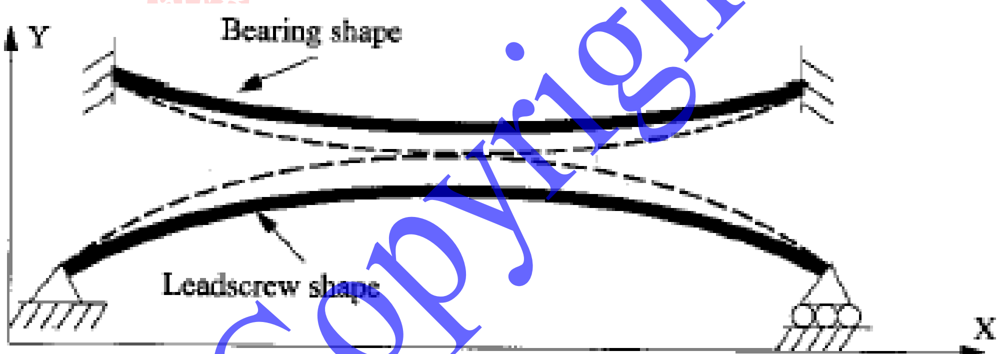


Elastic Averaging vs Exact Constraint Design

- Elastic averaging seems so contrary to *Exact-Constraint Design* that some people may argue one philosophy over the other rather than embracing their complementary virtues.
- Many kinematic designs rely on bearing systems that function by elastic averaging.
- Applying Reciprocity to *Exact Constraint Design* implies that instead of having an exact number of constraints, have an "infinite" number of constraints, so the error in any one will be averaged out!
- Overconstraint is NOT Elastic Averaging

Elastic Averaging vs Exact Constraint Design

- *Ex*: Often one component wants to move along one path and another along another, but they are attached to each other
  - Thus they will fight each other, and high forces can result which accelerates wear
  - Either more accurate components and assembly is required, or compliance, or clearance (pin in oversized hole) must be provided between the parts



Centers-of-Action: Outline

- ❒ Centers-of-Action
- ❒ Center-of-Mass
- ❒ Center-of-Stiffness
- ❒ Center-of-Friction
- ❒ Center-of-Thermal Expansion
- ❒ Robust Design

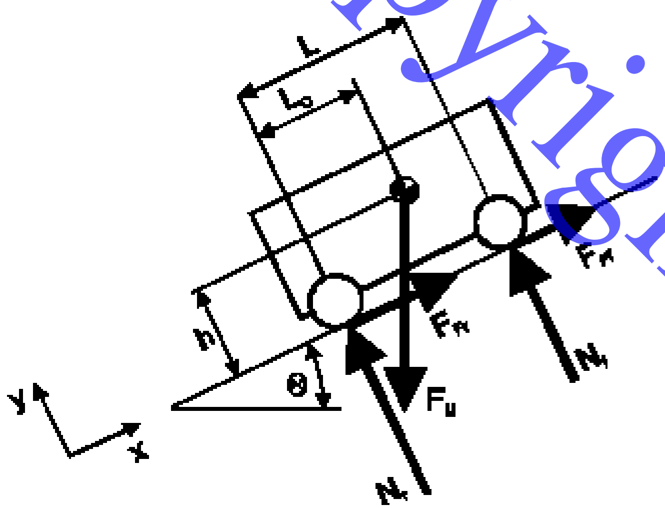
Introduction

- The ***Centers-of-Action*** are points at which when a force is applied, no moments are created
  - *Center-of-Mass*
  - *Center-of-Stiffness*
  - *Center-of-Friction*
  - *Center-of-Thermal Expansion*

Introduction

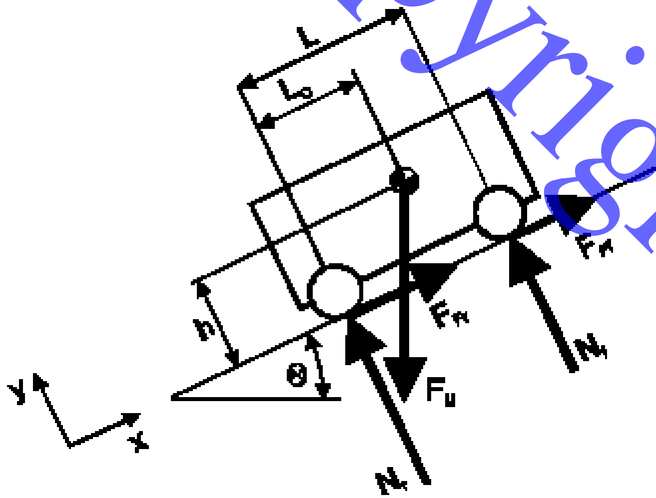
❒ **A system is most robust when forces are applied as near as possible to the *<u>Centers-of-Action</u>*.**

Center-of-Mass



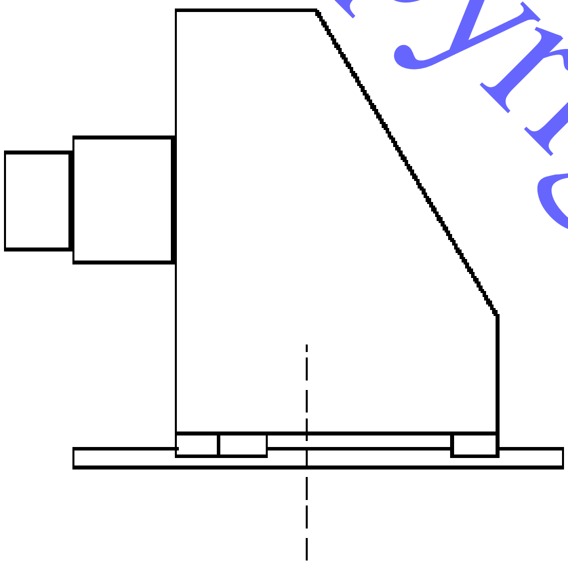
- The concept of the Center-of-Mass (or center-of-gravity, cg) is well known to most people
  - The center-of-mass of a system of particles moves like a single particle of mass $M=\Sigma m_i$ under the influence of the resultant external force acting on the system
  - …

Center-of-Mass



- The concept of the Center-of-Mass (or center-of-gravity, cg) is well known to most people
  - The center-of-mass is the point at which when a force is applied, an object undergoes only linear acceleration and thus has no angular acceleration component (which would otherwise lead to *Abbe* errors!)

Center-of-Stiffness



center of stiffness axis

$$X_{center\_of\_stiffness} = \frac{\sum_{i=1}^{N} X_i K_i}{\sum_{i=1}^{N} X_i}$$

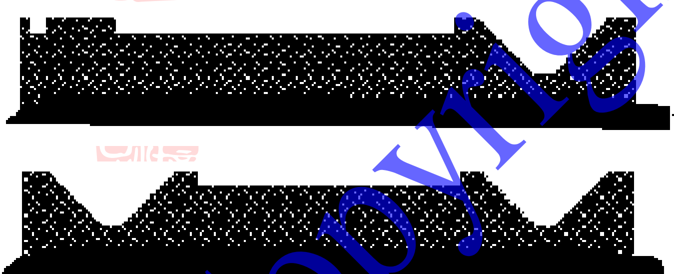
- A body supported by bearings, behaves as if all the bearings are concentrated at the ***center of stiffness***
  - The point at which when a force is applied to a locked-in-place axis, no angular motion of the structure occurs (which would otherwise lead to *Abbe* errors!)
  - It is also the point about which angular motion occurs when forces are applied elsewhere on the body

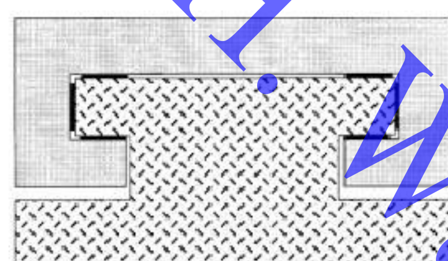
Center-of-Stiffness

- EXTREMELY POWERFUL PHILOSOPHICAL TOOL, AS IT ALLOWS A DESIGNER TO CREATE A MACHINE AS A STICK FIGURE
- Structure and bearings are added during embodiment phase so as to make the center-of-stiffness at the stick figure nodes

Center-of-Friction

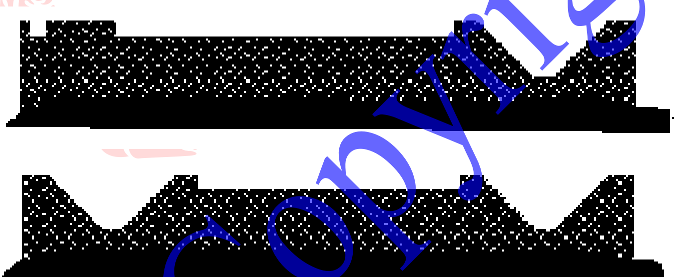
- A body supported by bearings, has friction forces in proportion to the weight distribution and the number and position of the bearings
- The <u>*center-of-friction*</u> is the point at which when a force is applied to a moving structure to which no other external forces are applied, no angular motion of the structure occurs (which would otherwise lead to *Abbe* errors!)

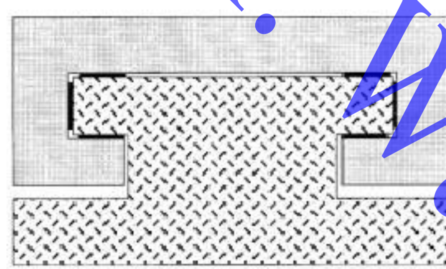




Center-of-Friction

- Found using force and moment balance equations that consider the effects of friction, bearing geometry, and center-of-gravity
  - The center-of-friction is sometimes, but not always, located at the center-of-stiffness
  - If a load is applied to different positions on a Vee-and-Flat and a Double-Vee supported carriage, how do the center-of-friction and the center-of-stiffness vary?





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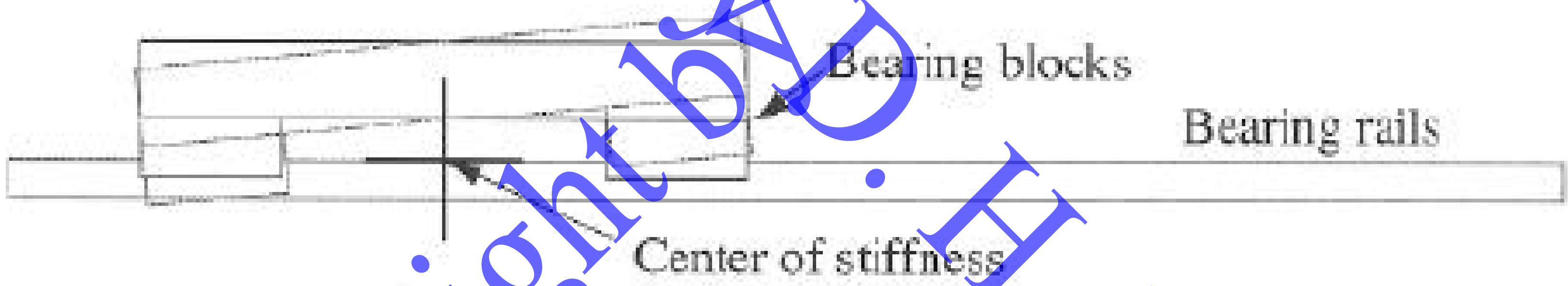
Center-of-Thermal Expansion

Robust Design

- ❒ If the primary work zone is near the center-of-mass, stiffness, and friction, errors will be minimized
- ❒ If the actuators for a machine apply their forces near the center-of-mass, stiffness, and friction, errors will be minimized
  - It is often difficult to make all three coincide, but get them as close as you can

Robust Design

- If a machine element (e.g., a leadscrew nut) is located at the center-of-stiffness, then error motions of one machine element (wobble of the screw) will not cause pitch errors (Abbe errors) in another element (carriage)



The End

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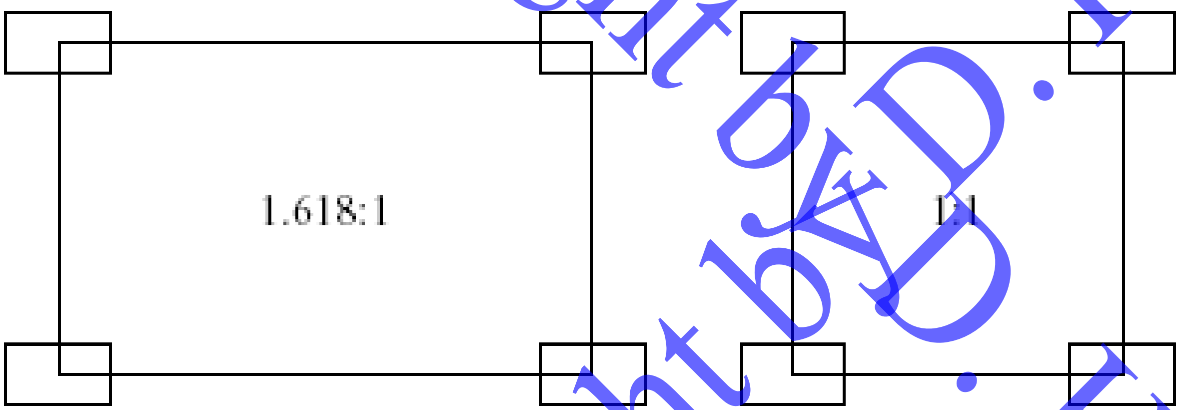
The Golden Rectangle: Outline

- ❒ **Definition**
- ❒ **Ex. Bearings**

Definition

- ***Golden Rectangle:*** **A rectangle where when a square is cut from the rectangle, the remaining rectangle has the same proportions as the original rectangle**



- **The proportions of the *Golden Rectangle* are a natural starting point for preliminary sizing of structures and elements**

Ex. Bearings

- The greater the ratio of the longitudinal to latitudinal (length to width) spacing:
  - The smoother the motion will be and the less the chance of walking (yaw error)
- First try to design the system so the ratio of the longitudinal to latitudinal spacing of bearing elements is about 2:1
- For the space conscious, the bearing elements can lie on the perimeter of a golden rectangle (ratio about 1.618:1)
- Next Slide…

The Golden Rectangle

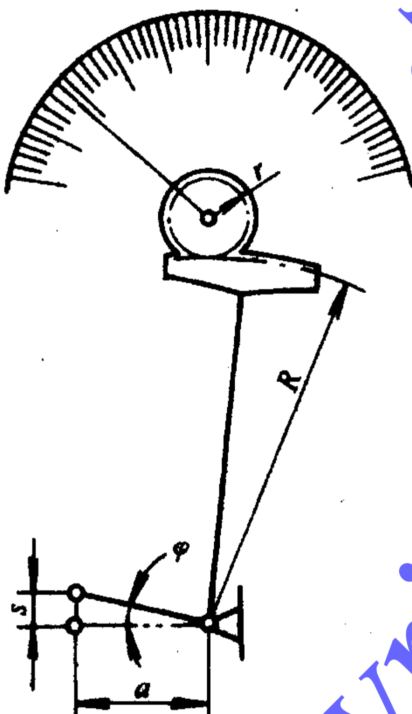
- The minimum length to width ratio is 1:1 to minimize yaw error
- The higher the speed, the higher the length to width ratio should be

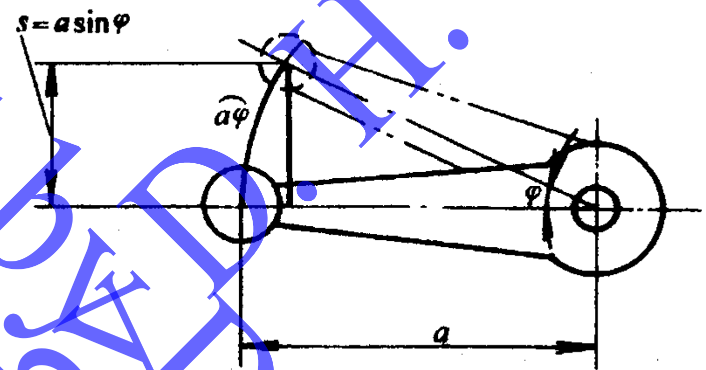
Compensating Principle: Outline

- ❒ Basic concept
- ❒ Key problems
  - Compensating tache
  - Compensating method
  - Compensating requirement
- ❒ *Ex*:
  - Compensating for Abbe error
  - Compensating for the force deformation
  - Micrometer (Optimal adjust principle)
  - Collimator

Optimal Adjust Principle: Dial Indicator

❐ 杠杆齿轮式测微仪

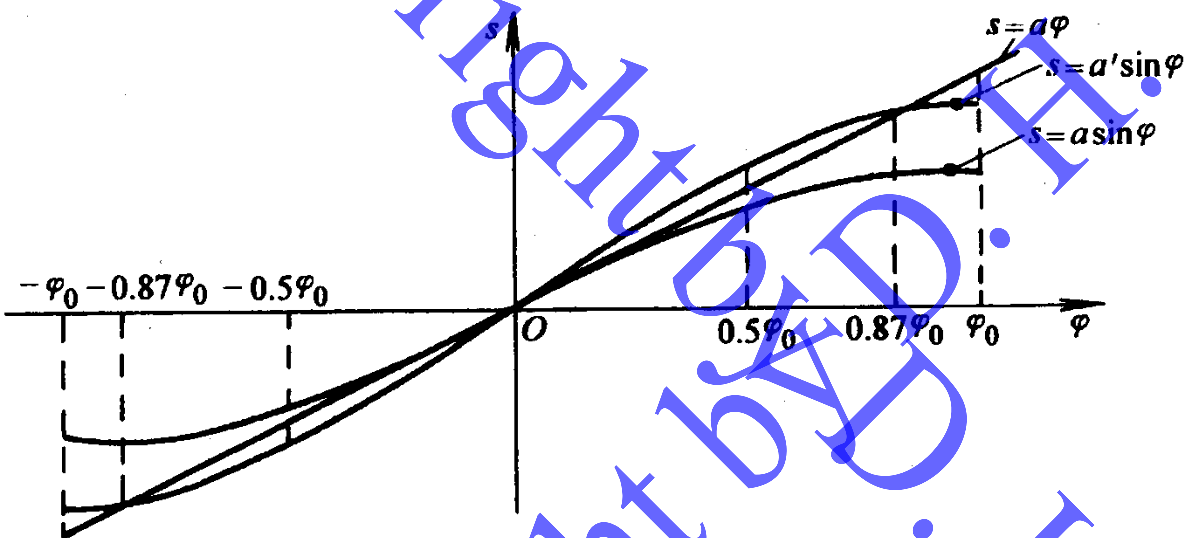




$$\left.\begin{aligned} s &= a\sin\varphi \\ s' &= a\varphi \end{aligned}\right\} \quad \Delta s = \frac{1}{6}a\varphi^3$$

Optimal Adjust Principle: Dial Indicator

❒ 调整方法

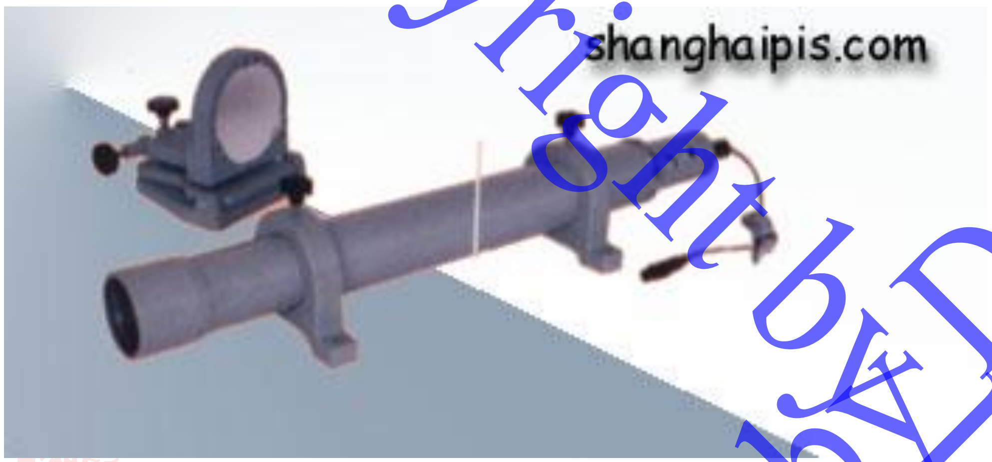


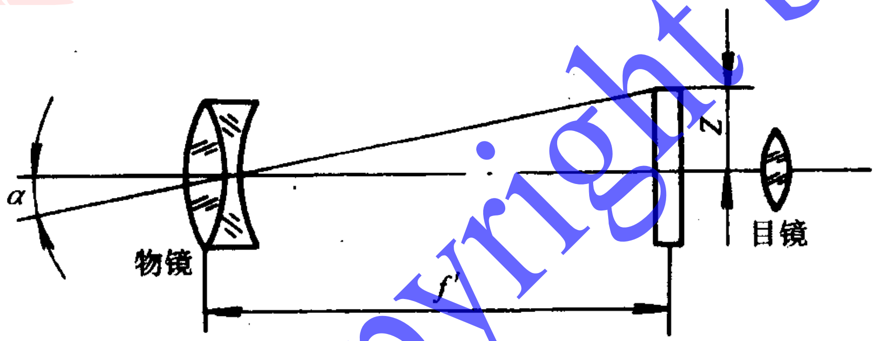
最佳调整： $\varphi = 0.874\varphi_0\big|_{\Delta s=0}$ $\Delta s = \frac{1}{24} a\varphi^3$

$$a' = \frac{0.87 a\varphi_0}{\sin(0.87\varphi_0)}$$

Optimal Adjust Principle: Collimator

- Collimator and its working principle





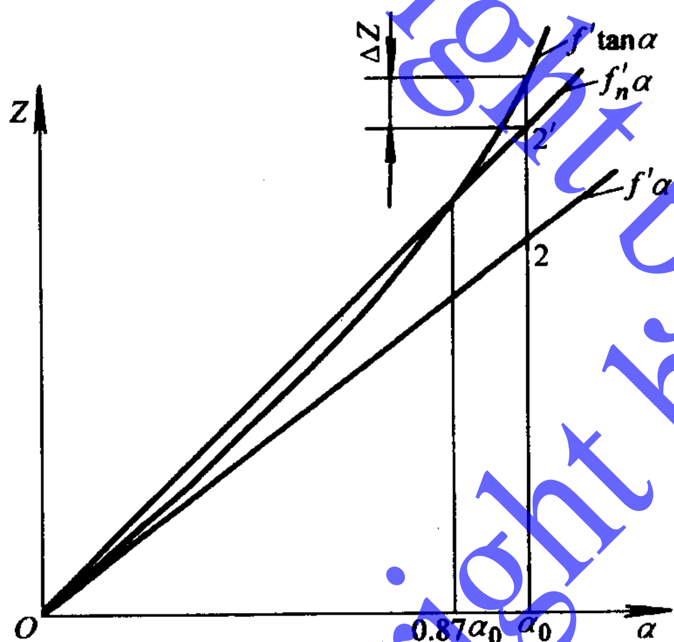
$$Z = f^{'} \tan \alpha$$

$$Z^{'} = f^{'} \alpha$$

$$\Delta Z = Z - Z^{'} = -\frac{1}{3} f^{'} \alpha^{3}$$

Optimal Adjust Principle: Collimator

❒ **Optimal adjustment principle**



$$\Delta Z\Big|_{\alpha=0.87\alpha_0} = 0$$

$$f_n' = \frac{f' \tan 0.87\alpha_0}{0.87\alpha_0}$$

Summary

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The End

Thank you very much for your attention!

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勿以善小而不为，
勿以恶小而为之！

三国志.陈寿