Chapter 4. Precision Machine System Design

Lecture 3. Slides and Design

王代华，博士，教授，博士生导师

Dr. Dai-Hua, Wang, Professor

PILab 精密與智能實驗室 Precision and Intelligence Laboratory

http://www.pilab.coe.cqu.edu.cn/

Email: dhwang@cqu.edu.cn

Tel: 023-65112105(O), 65102511(Lab)

重庆大学，光电工程学院

Outline

- ❒ Usage
- ❒ Types
- ❒ Design Requirements
- ❒ Principles to be Abided By
- ❒ Slide Contact Linear Bearings
- ❒ Rolling Element Linear Motion Bearings
- ❒ Hydrostatic Bearings
- ❒ Aerostatic Bearing
- ❒ Hydro-dynamic Bearings
- ❒ Flexural Bearings

Usage

Types

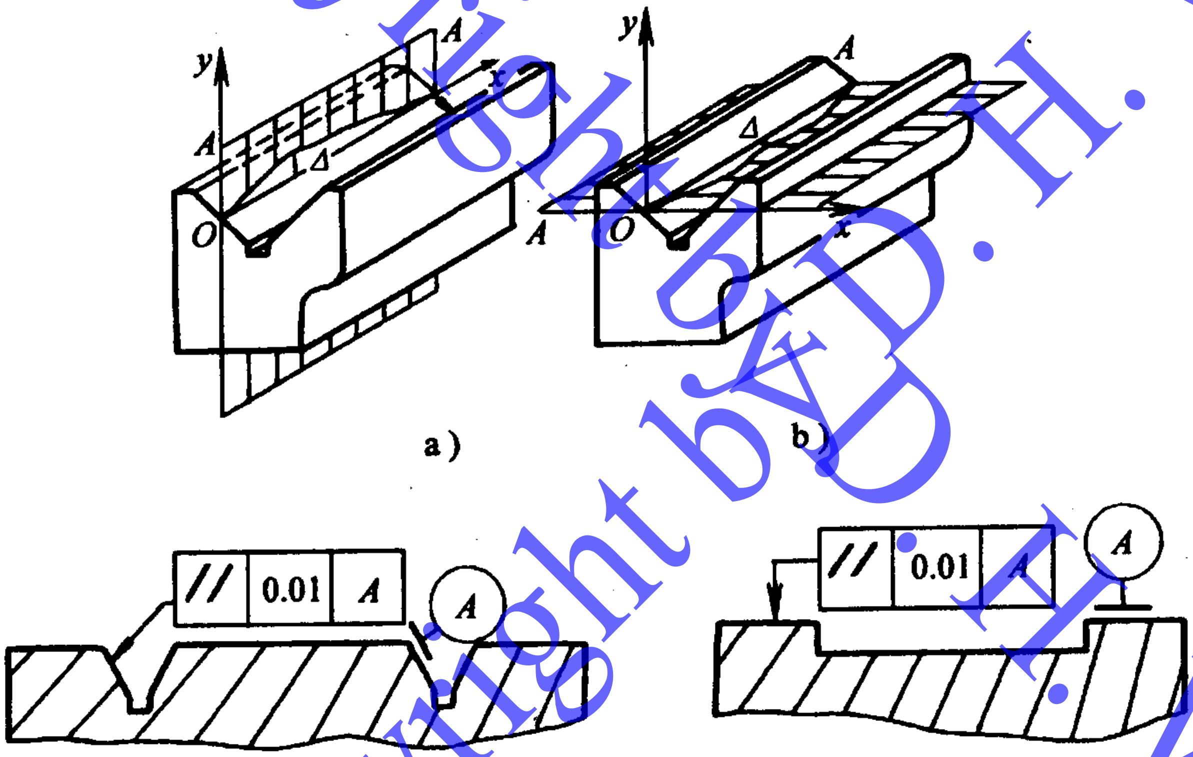
- ❒ **Slide contact linear bearings**
- ❒ **Rolling element linear motion bearings**
- ❒ **Hydrostatic bearings**
- ❒ **Aerostatic bearing**
- ❒ **Hydro-dynamic bearings**
- ❒ **Flexural bearings**

Outline

- ❒ **Accuracy**
- ❒ **Stability**
- ❒ **Stiffness**
- ❒ **Anti-wear**

Accuracy

- **Geometrical errors**



a)

b)

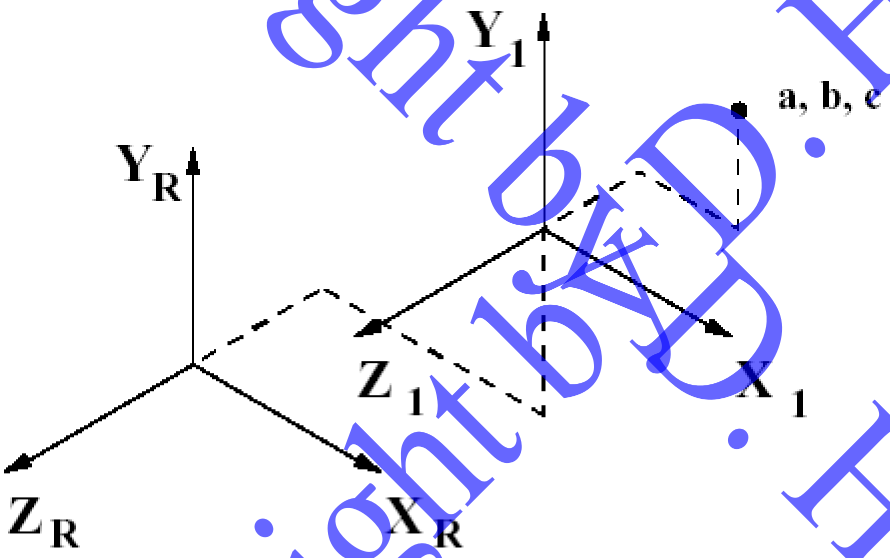
c)

Accuracy

- 接触精度

Accuracy

- ❐ Homogeneous transformation matrices for error assessment and budgeting



Accuracy

- **Structure of a homogeneous transformation matrix**
  - **The equivalent coordinates of a point in a reference frame n, in a reference frame R are**

$$\begin{bmatrix} X_R \\ Y_R \\ Z_R \\ 1 \end{bmatrix} = {}^R\mathbf{T}_n \begin{bmatrix} X_n \\ Y_n \\ Z_n \\ 1 \end{bmatrix} \qquad {}^R\mathbf{T}_n = \begin{bmatrix} O_{ix} & O_{iy} & O_{iz} & P_x \\ O_{jx} & O_{jy} & O_{jz} & P_y \\ O_{kx} & O_{ky} & O_{kz} & P_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

  - **Transformation from the Nth axis to the reference system will be the sequential product of all the HTMs**
  - **The relative error HTM Erel between the tool and workpiece in the tool coordinate frame is**

Accuracy

- Structure of a homogeneous transformation matrix
  - The equivalent coordinates of a point in a reference frame n, in a reference frame R are
  - Transformation from the Nth axis to the reference system will be the sequential product of all the HTMs

$$ {}^{R}\mathbf{T}_{N} = \prod_{m=1}^{N} {}^{m-1}\mathbf{T}_{m} = {}^{0}\mathbf{T}_{1}\, {}^{1}\mathbf{T}_{2}\, {}^{2}\mathbf{T}_{3} \, . \, . \, . \, . $$

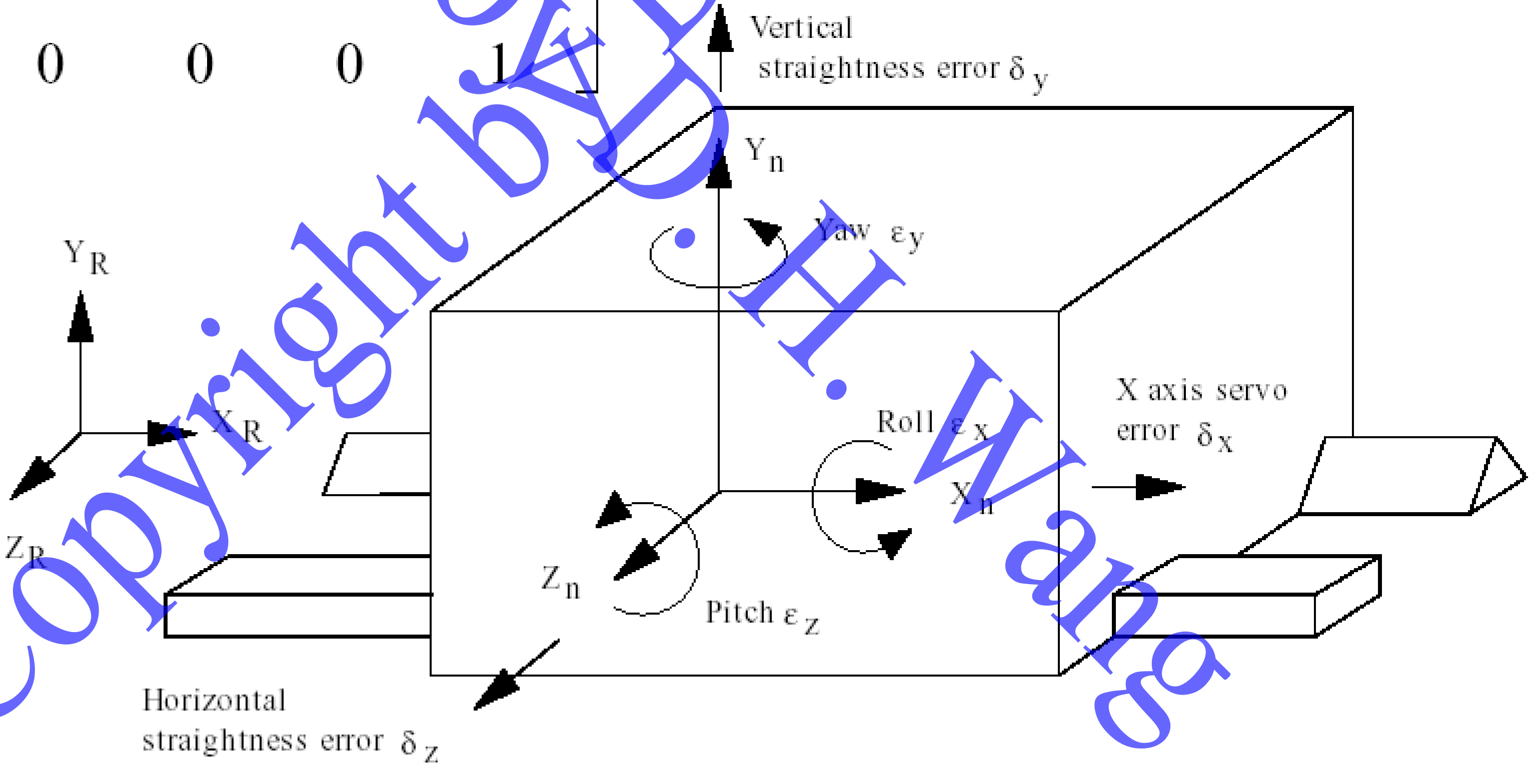
  - The relative error HTM Erel between the tool and workpiece in the tool coordinate frame is

$$ E_{rel} = {}^{R}\mathbf{T}_{tool}{}^{-1}\, {}^{R}\mathbf{T}_{work} $$

Accuracy

❐ **The HTM for a linear motion carriage with small errors**

$$
{}^{R}\mathbf{T}_{nerr} = \begin{bmatrix} 1 & -\varepsilon_Z & \varepsilon_Y & a+\delta_X \\ \varepsilon_Z & 1 & -\varepsilon_X & b+\delta_Y \\ -\varepsilon_Y & \varepsilon_X & 1 & c+\delta_Z \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

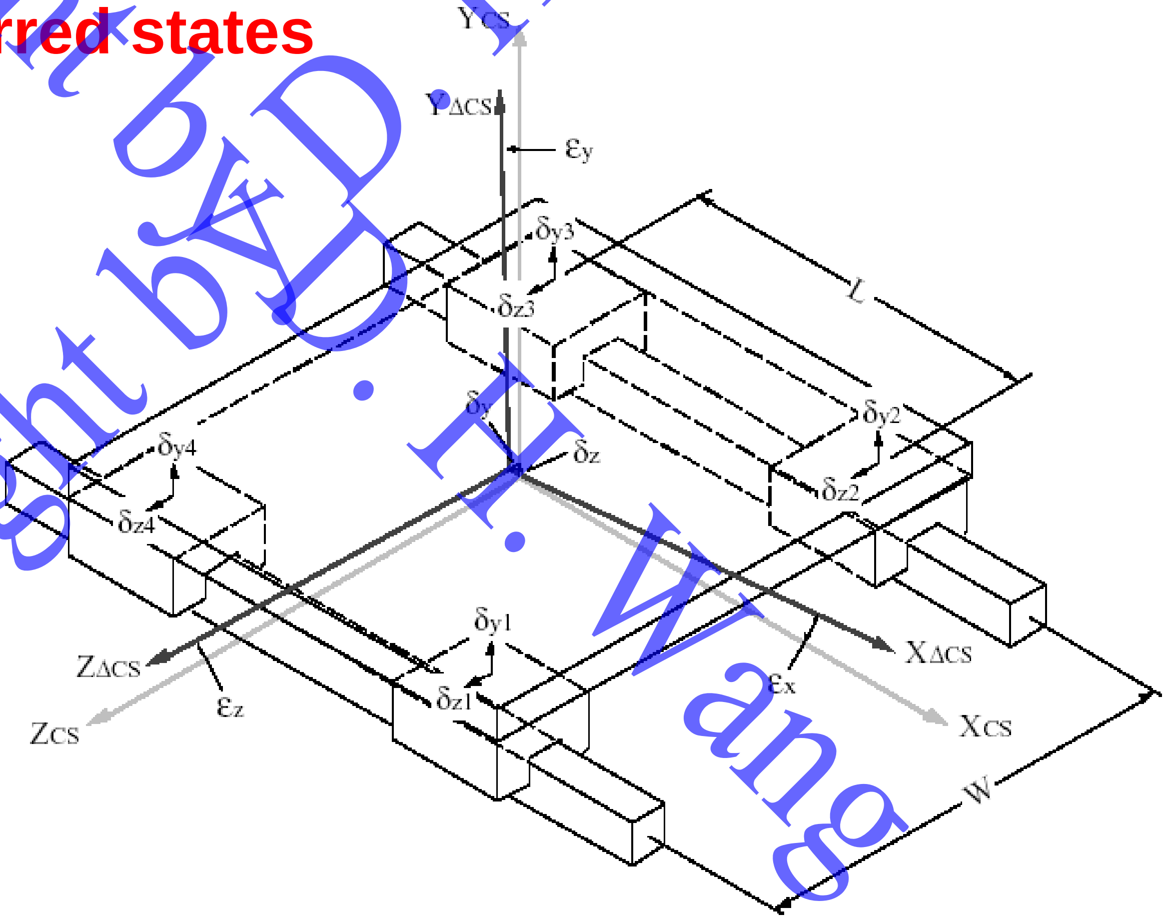


Accuracy

- ❒ **Estimating position errors from modular bearing catalog straightness data**
  - **The HTM method is powerful, but from where does one get estimates of the errors?**
  - **The HTM assumes that the errors occur at the center of stiffness of the carriage.**
    - **The center of stiffness is the point at which when a force is applied to the system, no net angular motion results.**

Accuracy

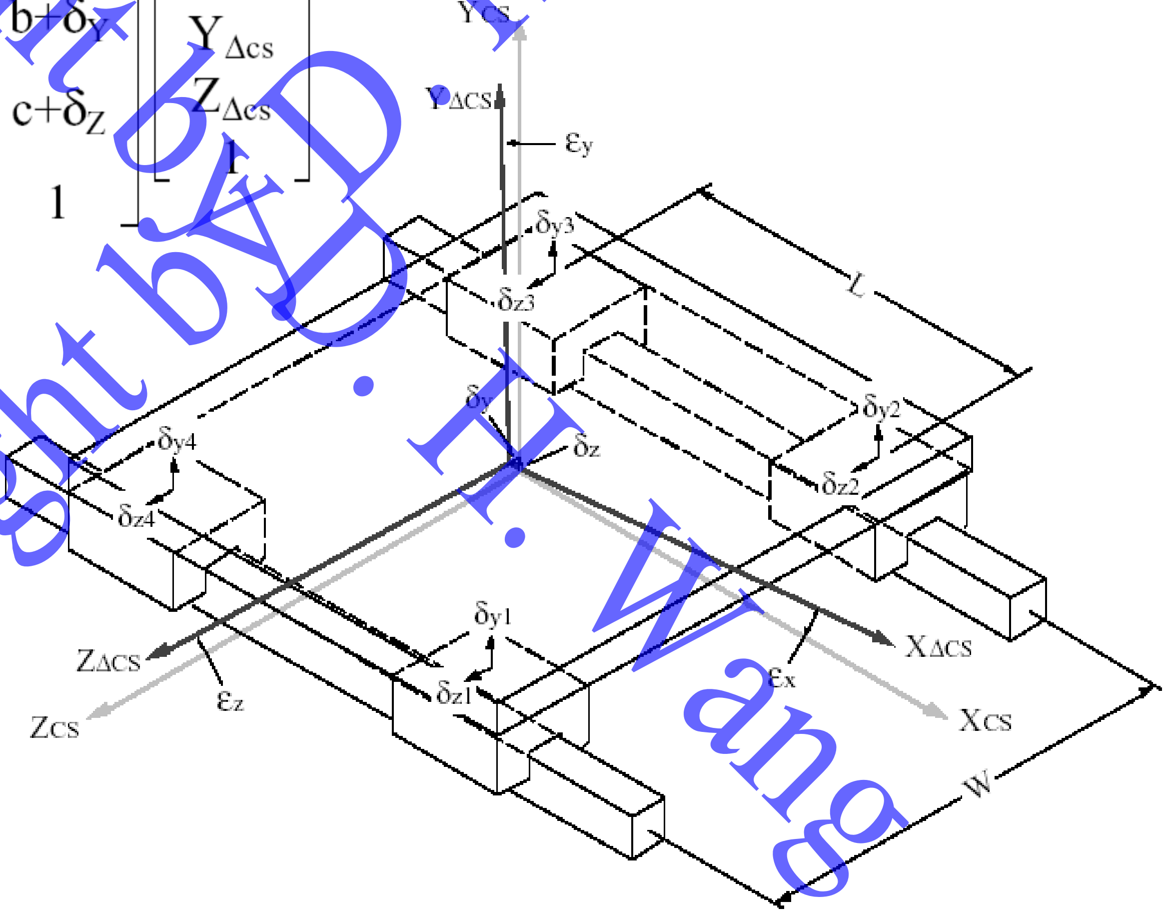
- ❒ **Estimating position errors from modular bearing catalog straightness data**
  - ✐ **The first step is to draw a sketch of the system in the ideal and erred states**



Accuracy

- ❒ Consider the elements of the HTM we are searching for

$$\begin{bmatrix} X_{cs} \\ Y_{cs} \\ Z_{cs} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & -\varepsilon_Z & \varepsilon_Y & a+\delta_X \\ \varepsilon_Z & 1 & -\varepsilon_X & b+\delta_Y \\ -\varepsilon_Y & \varepsilon_X & 1 & c+\delta_Z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_{\Delta cs} \\ Y_{\Delta cs} \\ Z_{\Delta cs} \\ 1 \end{bmatrix}$$



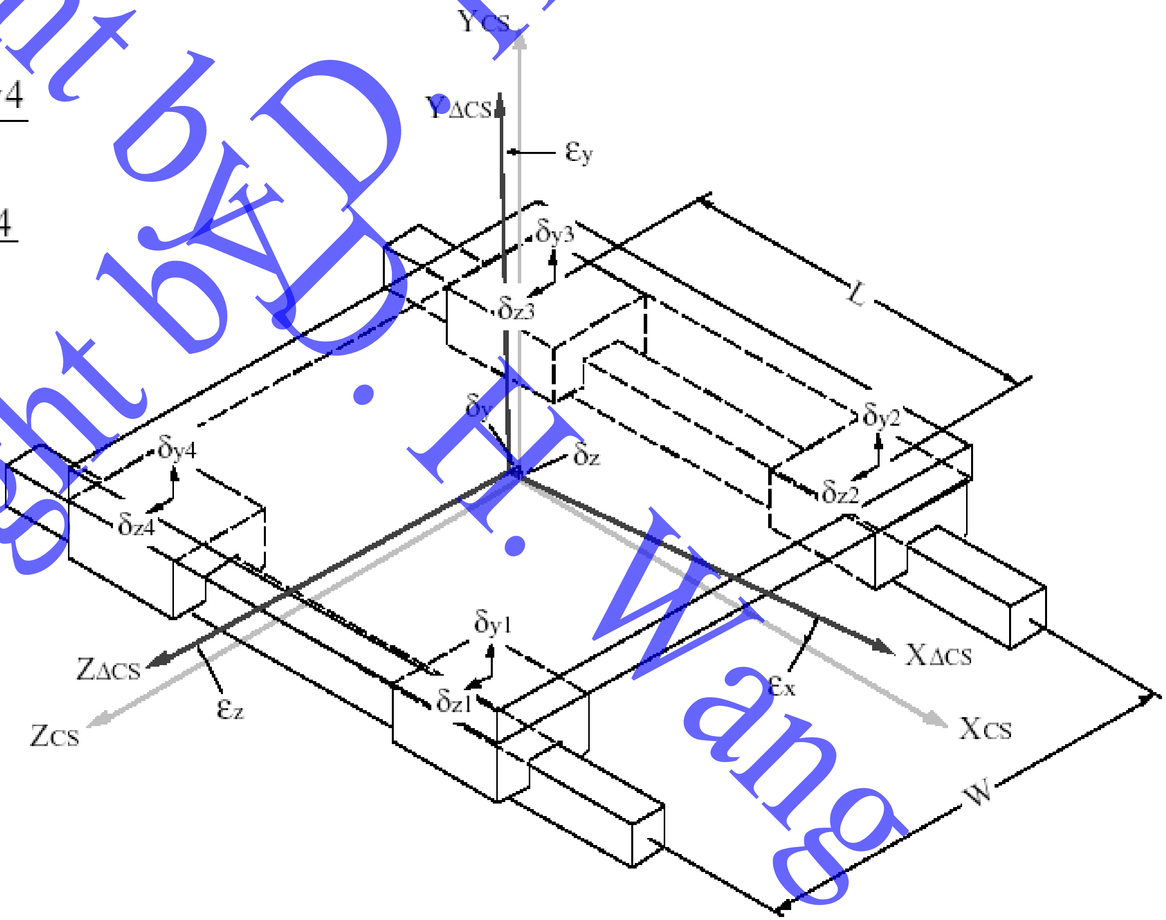
Accuracy

- The translational errors are based on the average of the straightness errors experienced by the bearing blocks

$$\delta_x = \delta_{servo}$$

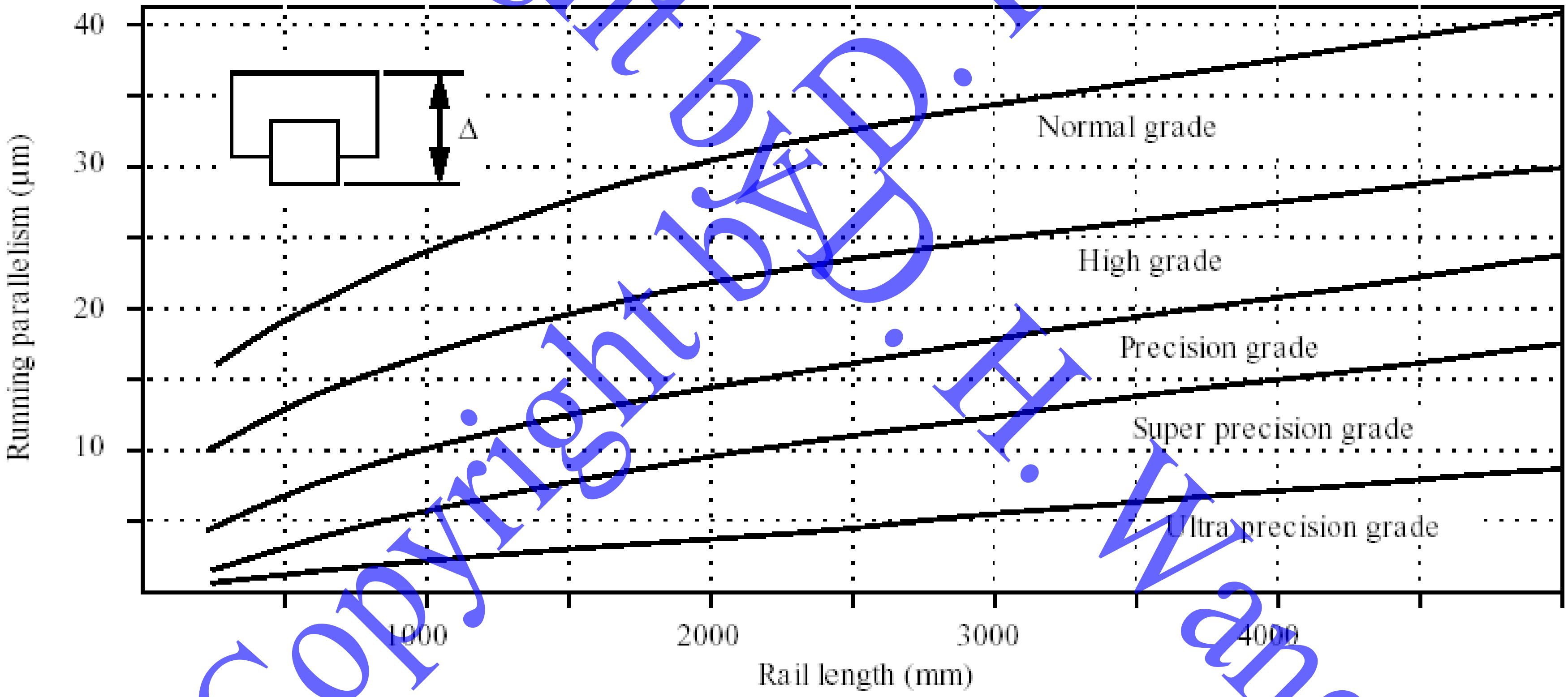
$$\delta_y = \frac{\delta_{y1} + \delta_{y2} + \delta_{y3} + \delta_{y4}}{4}$$

$$\delta_z = \frac{\delta_{z1} + \delta_{z2} + \delta_{z3} + \delta_{z4}}{4}$$



Accuracy

- Bearing block straightness is a function of bed accuracy and running parallelism of the bearing block to the bearing rail



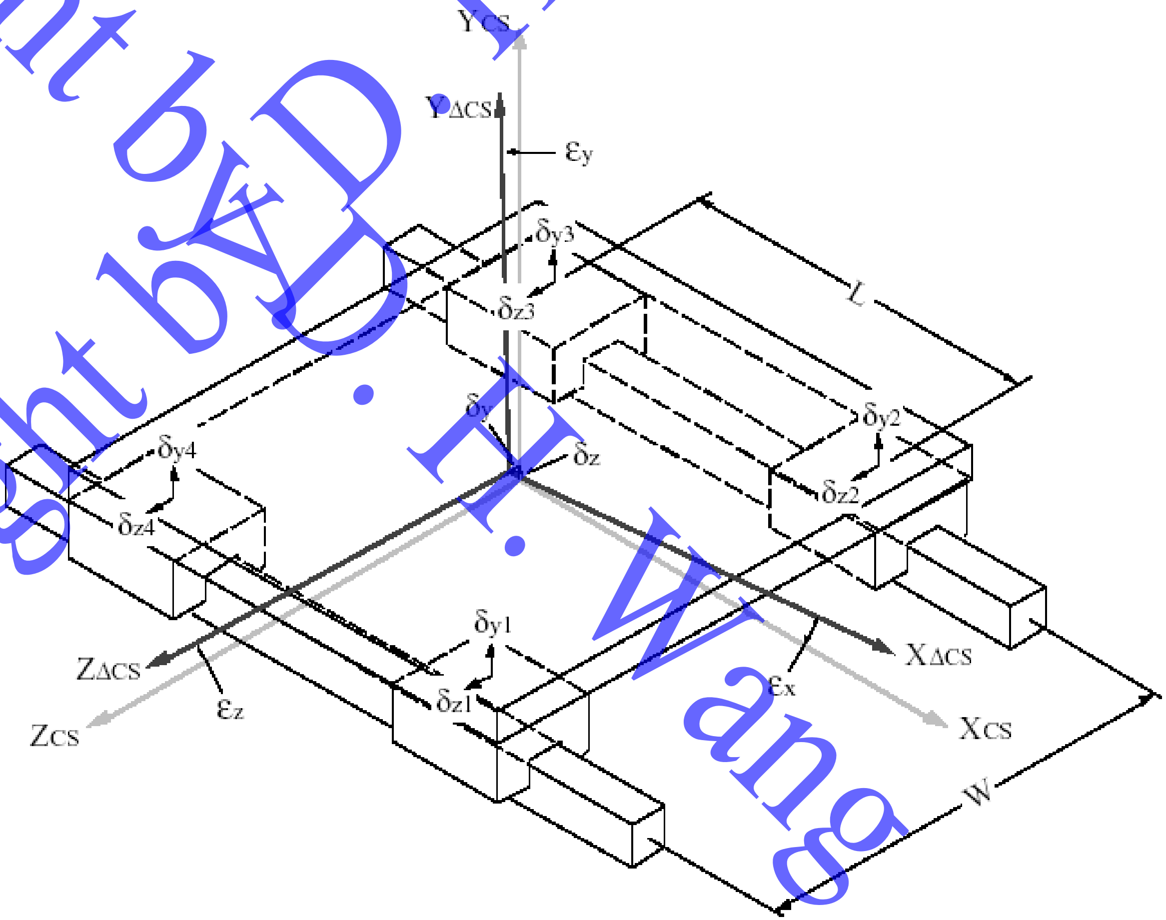
Accuracy

- ❒ The angular errors are based on the differences in the average straightness errors experienced by pairs of bearing blocks acting across the carriage

$$\varepsilon_x = \frac{\dfrac{(\delta_{y2}+\delta_{y3})}{2} - \dfrac{(\delta_{y1}+\delta_{y4})}{2}}{W}$$

$$\varepsilon_y = \frac{\dfrac{(\delta_{z3}+\delta_{z4})}{2} - \dfrac{(\delta_{z1}+\delta_{z2})}{2}}{L}$$

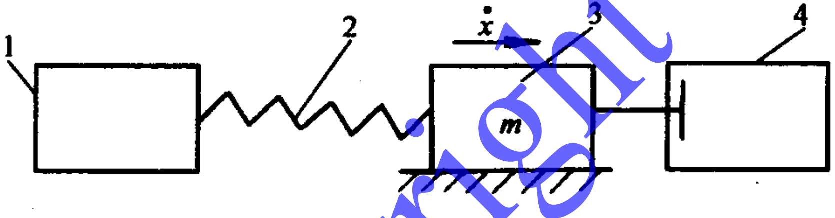
$$\varepsilon_z = \frac{\dfrac{(\delta_{y1}+\delta_{y2})}{2} - \dfrac{(\delta_{y3}+\delta_{y4})}{2}}{L}$$



Stick-Slip (Stiction)

❐ **Causes of stick-slip**

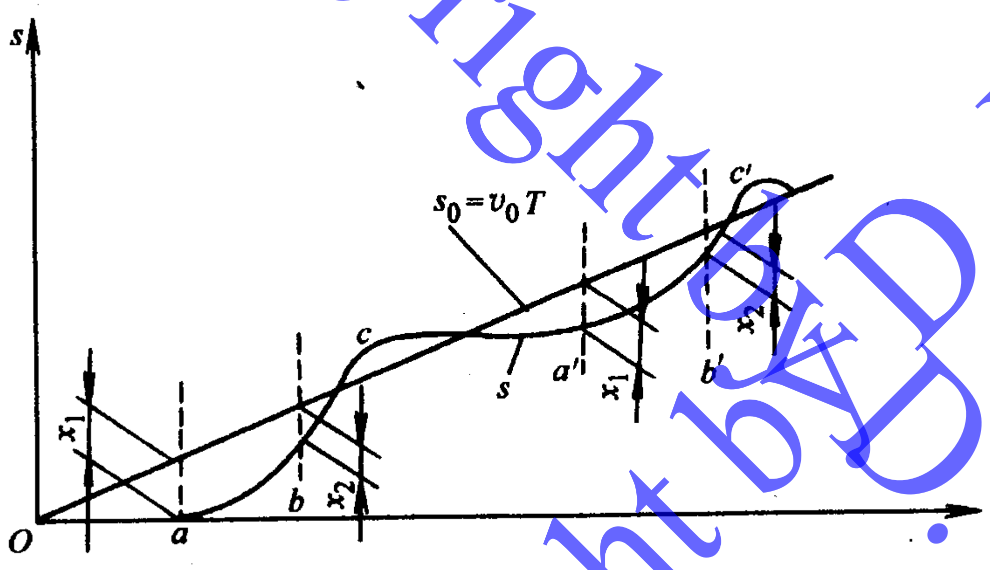
- The difference between the static friction coefficient and the dynamic friction coefficient
- The dynamic friction coefficient variation with the velocity
- Lower stiffness



Mechanical Model of Linear Motion

Stick-Slip (Stiction)

- ❐ **Critical velocity**



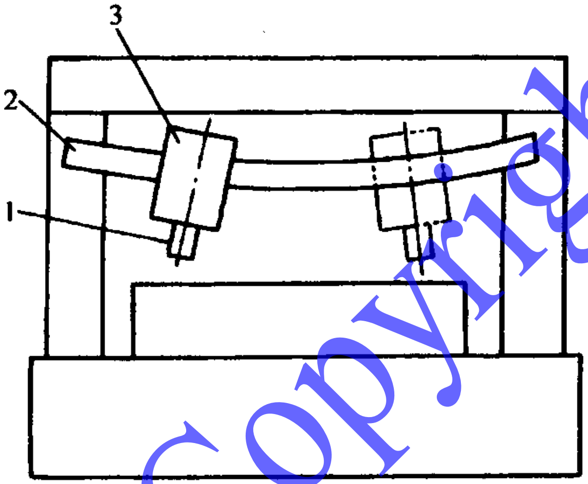
$$v_c = \frac{\Delta F}{\sqrt{4\pi\xi mK}}$$

Mechanical Model of Stick-Slip

Stiffness

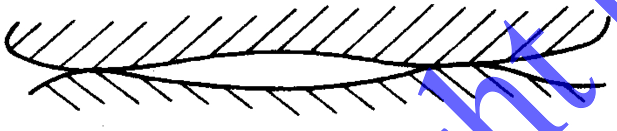
□ Weight induced deformation

- 自重变形是作用在导轨上零部件的重量造成的。
- 减少自重变形的办法
  - 采用刚度设计，如有限元法；
  - 结构设计，如设计加强肋；
  - 补偿措施，如用螺钉或其他方法反变形。



Stiffness

- ❒ **Local deformation and contact deformation**
  - 局部变形发生在载荷集中的地方，如立柱与导轨接触部分。
  - 接触变形是由于微观不平度造成实际接触面积是名义接触面积的很小一部分。

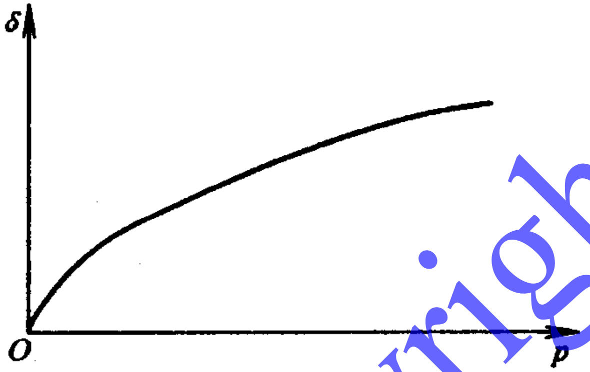


Contact Deformation

Stiffness

□ 为了减少接触变形，可以采用预加载荷的办法增加接触刚度

- 对于固定不动的接触面，预加载荷一般大于活动件及其上的部件的重力与外载荷的和；
- 对于活动的接触面，预加载荷一般等于活动件及其上工件等的重力和。



Relationship between pressure and deformation

$$\delta = \frac{p}{K_j}$$

Decreasing Wear

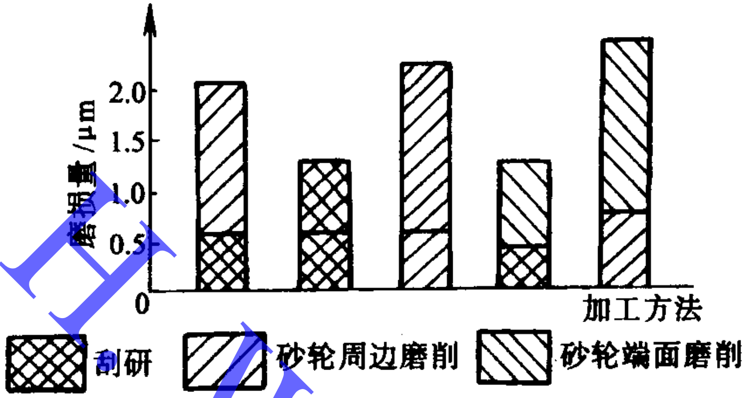
- ❒ Friction properties

$$p_s = \frac{W}{S} = \frac{W}{Bl}$$

- ❒ Load per unit area
- ❒ Good lubricity
- ❒ Material combinations
- ❒ Manufacturing methods
- ❒ Next slide…

Decreasing Wear

- Material combinations
  - It was found that by making one surface in a sliding contact bearing harder than the other, better wear characteristics were generally obtained. (1.1~1.2)
- Cases
  - Cast iron on cast iron
  - Cast iron on steel
  - Brass on steel
  - Polymers on most anything
  - Almost anything on ceramic
  - Manufacturing methods

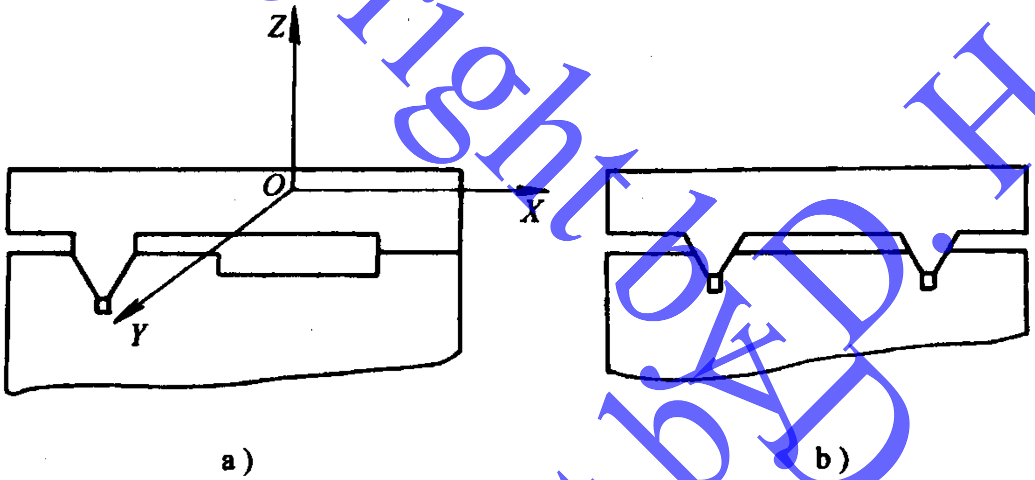


Outline

- ❒ Exact Constraint Design
- ❒ Elastic Averaging
- ❒ Separation the Slide Guide and the Preloaded Slide

Exact Constraint Design

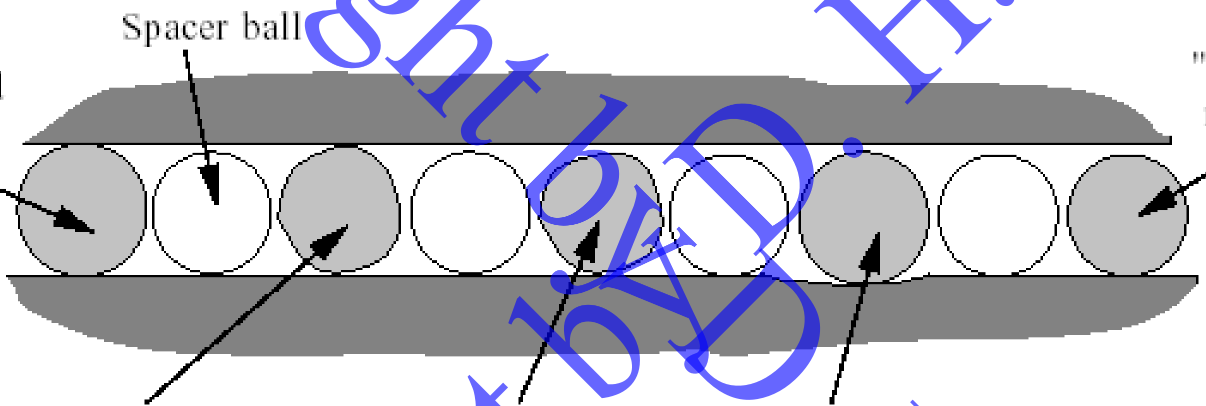
- ❒ Exact Constraint Design



- ❒ Elastic Averaging
- ❒ Separation the Slide Guide and the Preloaded Slide

Elastic Averaging

- ❒ Exact Constraint Design
- ❒ Elastic Averaging



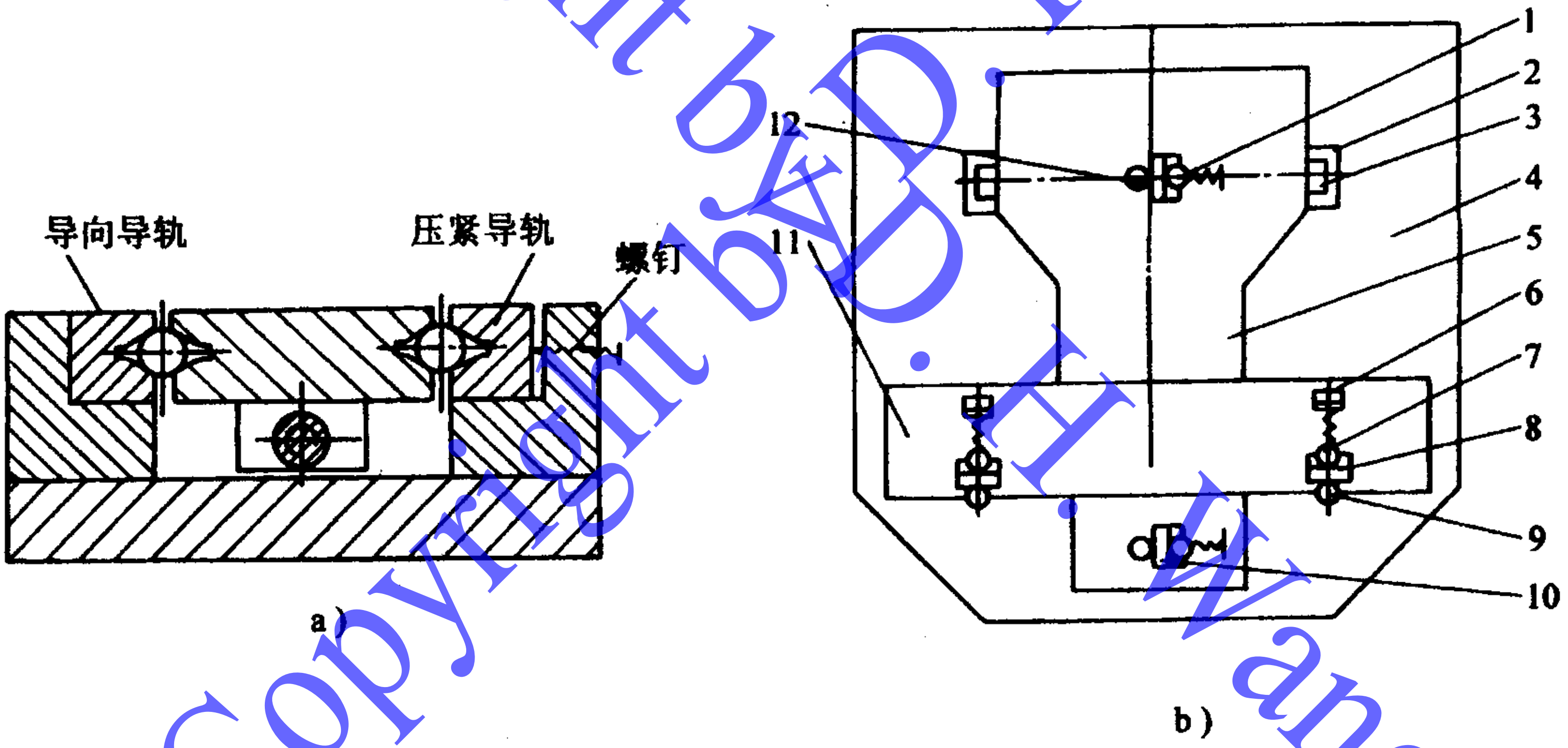
- ❒ Separation the Slide Guide and the Preloaded Slide

Separation the Slide guide and the Preloaded Slide

❒ Exact Constraint Design

❒ Elastic Averaging

❒ Separation the Slide Guide and the Preloaded Slide



Acknowledgement

Thank you very much for your attention !

E-mail: dhwang@cqu.edu.cn
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Types of Slides

- ❒ Slide contact linear bearings
- ❒ Rolling element linear motion bearings
- ❒ Hydrostatic bearings
- ❒ Aerostatic bearing
- ❒ Hydro-dynamic bearings
- ❒ Flexural bearings

Outline

- ❒ **Characteristics**
- ❒ **Configurations**

Characteristics

- ❒ General characteristics
  - ✐ Sliding contact are the "original" machine tool bearings.
  - ✐ They are very robust and reliable.
  - ✐ They are speed limited and have friction-induced servo limits.
  - ✐ They are economical and for many applications will never be replaced.
- ❒ Speed and acceleration limits
  - ✐ < 15 m/min (600 ipm) and 0.1 g.

Characteristics

- ☐ **Applied loads**
  - Large surface area allows for high load capacity.
  - Virtually insensitive to crashes
- ☐ **Accuracy**
  - Axial: 5 - 10 microns depending on the drive system.
  - Lateral (straightness): 0.1 - 10 microns depending on the rails.
  - Special designs can yield nanometer accuracy.
- ☐ **Repeatability**
  - Axial: 2 - 10 microns depending on the drive system.
  - Lateral (straightness): 0.1 - 10 microns depending on the rails.

Characteristics

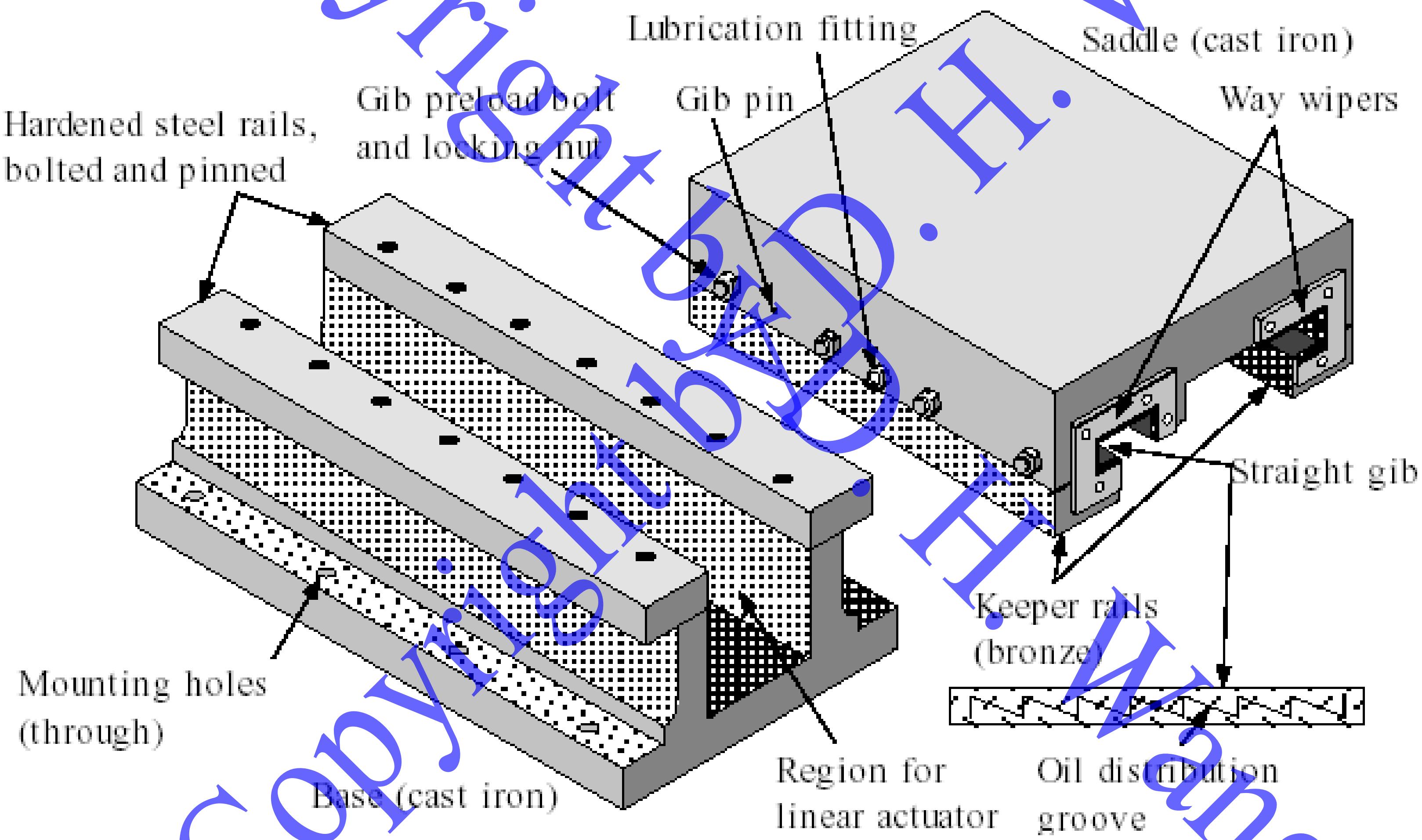
- ❒ **Resolution**
  - ✐ **Axial: 1 - 10 microns depending on the drive system.**
- ❒ **Preload**
- ❒ **Stiffness**
  - ✐ **5-10% of the allowable load.**
  - ✐ **Easily made many times greater than other components in the machine.**
  - ✐ **Stiffness of various sliding contact bearings lubricated with light oil and after wear-in.**

Characteristics

- ❒ **Friction**
  - **Static friction never equals dynamic friction.**
  - **Stiction, when static μ is greater than dynamic μ, cause stickslip which causes position errors.**
- ❒ **Vibration and shock resistance**
- ❒ **Damping capability**
- ❒ **Thermal performance**
- ❒ **Environmental sensitiveness**
- ❒ **Support equipment**
- ❒ **Maintenance**

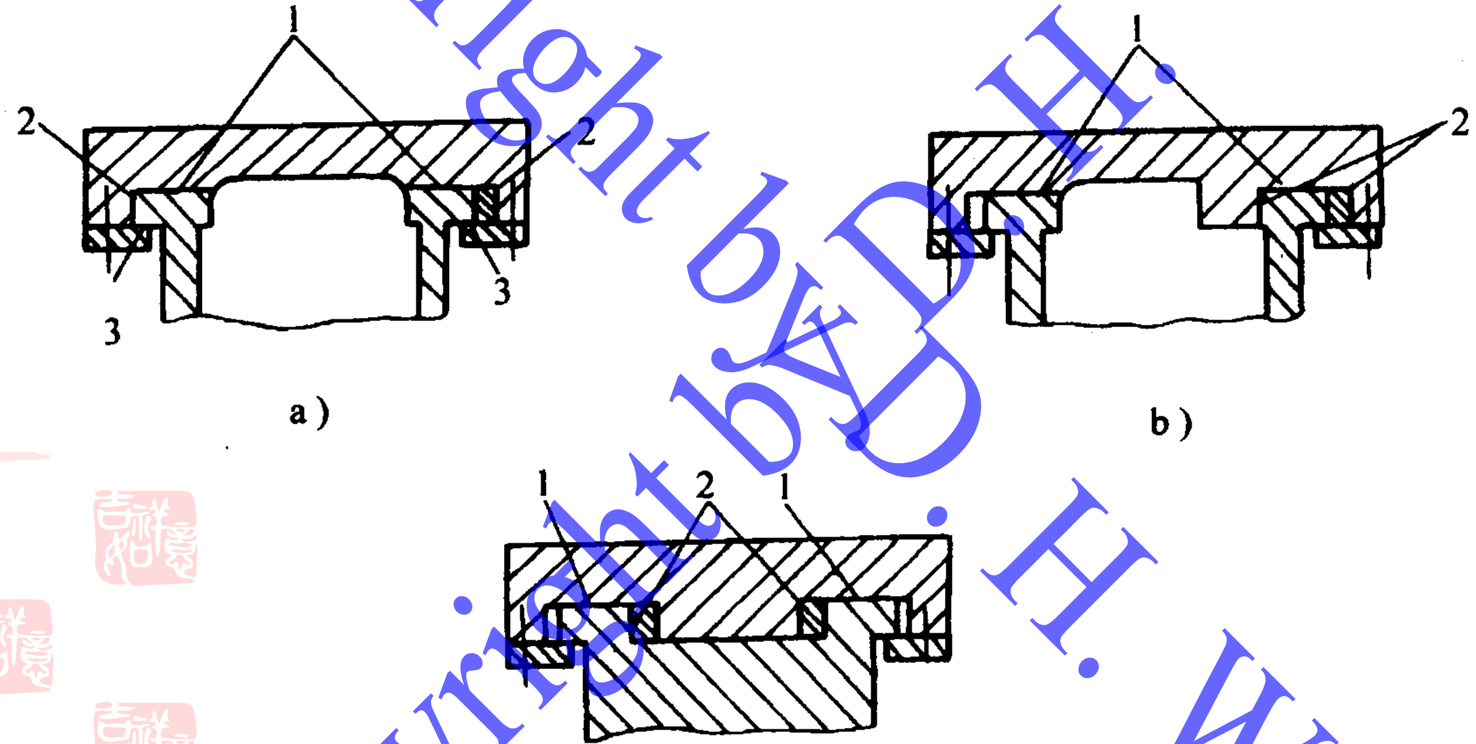
Configurations

- Open rectangular linear bearing configuration



Configurations

- ❒ Open rectangular linear bearing configuration



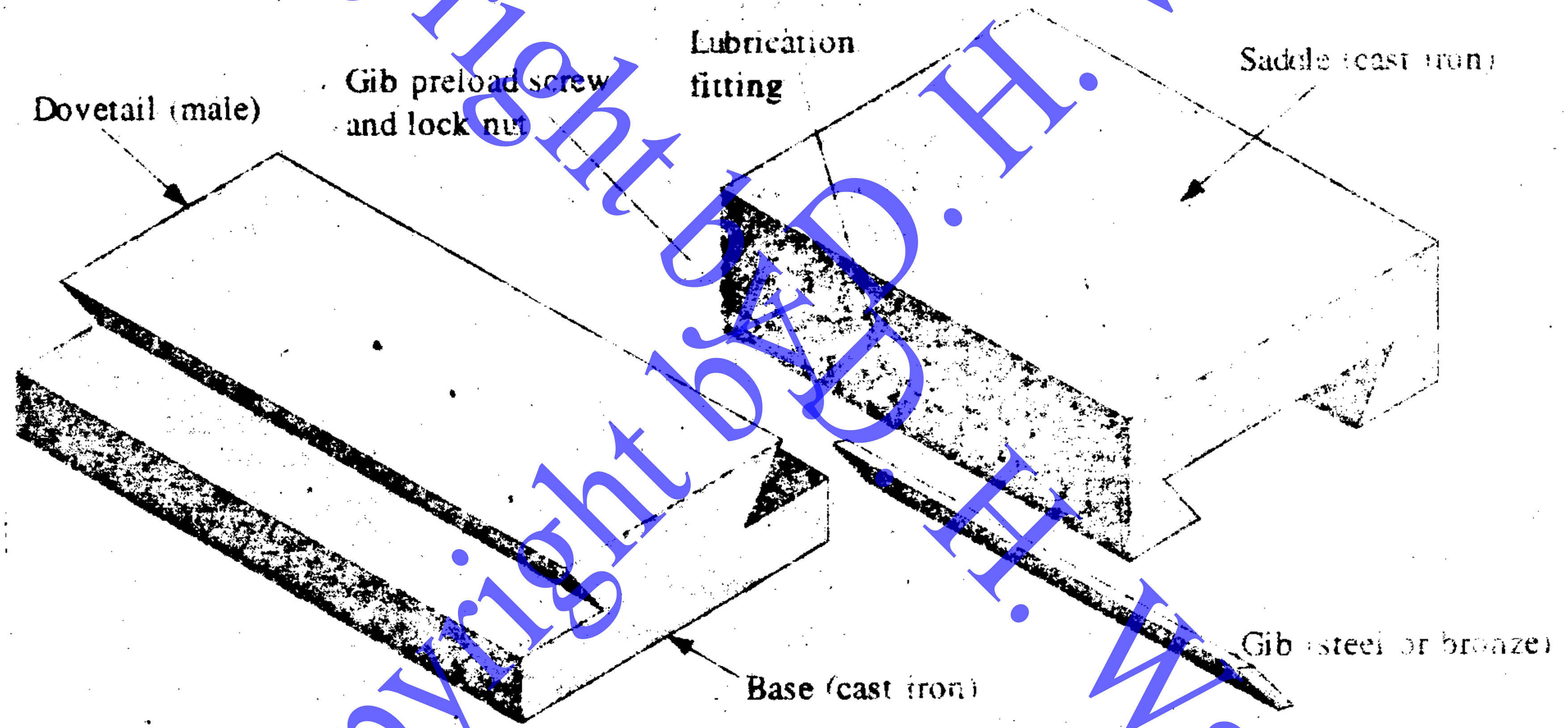
a)

b)

c)

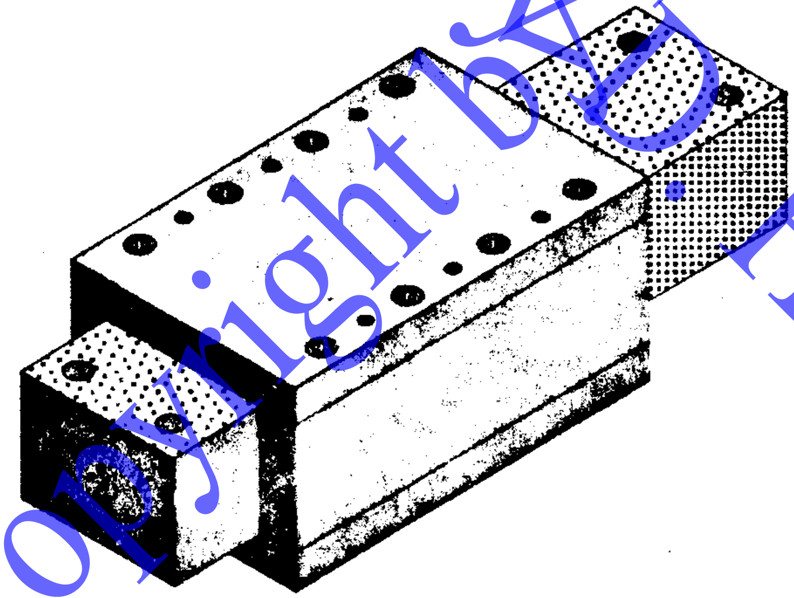
Configurations

❒ Construction of a modular dovetail slide assembly



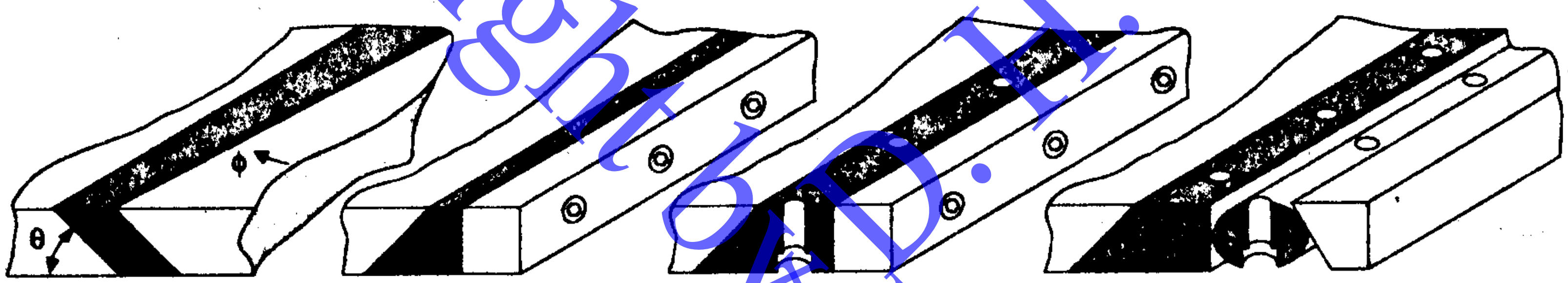
Configurations

- ☐ **Closed rectangular linear bearing configuration**
  - General configuration of a rectangular linear motion sliding contact bearing.
  - Bearing surface may be composed of pads or be the entire interface (use of gibs not shown)



Gibs

- Some of the many types of gibs that can be used to preload bearings



Tapered gib with in-line screw preload

Straight gib with setscrew preload

Straight gib with locking bolts setscrew preload

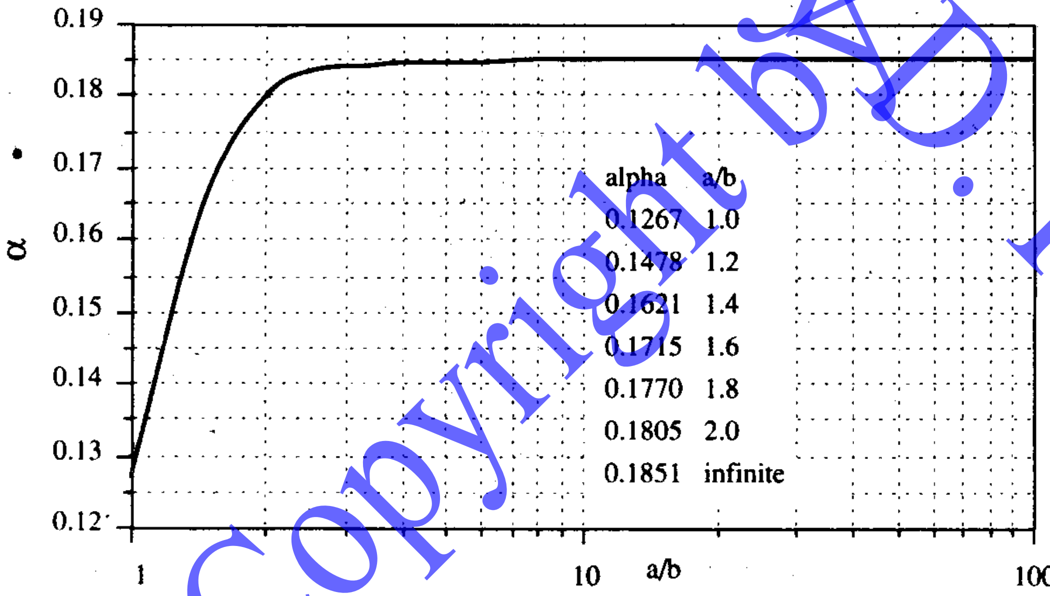
Straight gib with locking bolts and roller/wedge preload

- Model for calculating straight gib thicknesses

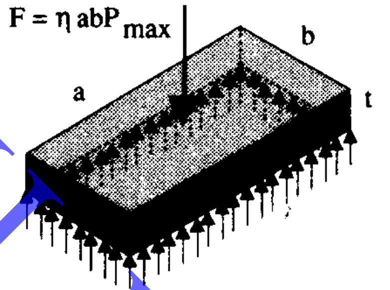
Gibs

- ❐ Some of the many types of gibs that can be used to preload bearings
- ❐ Model for calculating straight gib thicknesses

$$t_{gib} = \left(\frac{2\alpha \eta a b^3 P_{\max}}{\delta E}\right)^{1/3}$$



alpha	a/b
0.1267	1.0
0.1478	1.2
0.1621	1.4
0.1715	1.6
0.1770	1.8
0.1805	2.0
0.1851	infinite



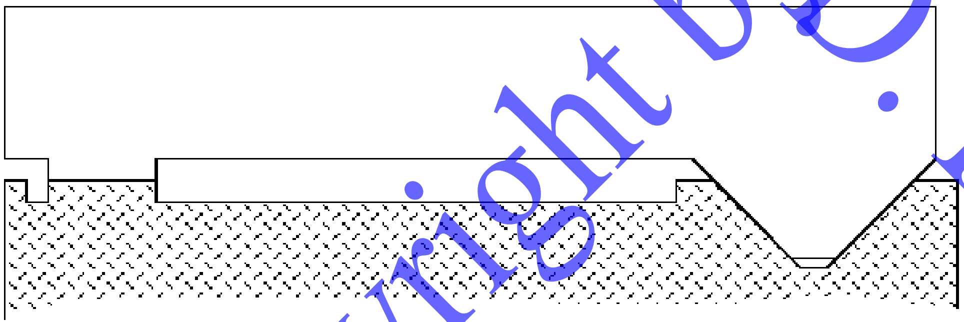
Uniformly supported around perimeter

$$\eta = \frac{\%\ \text{max rated load}}{100}$$

$$\delta = \frac{\alpha\eta\, ab^3 P_{max}}{Et^3}$$

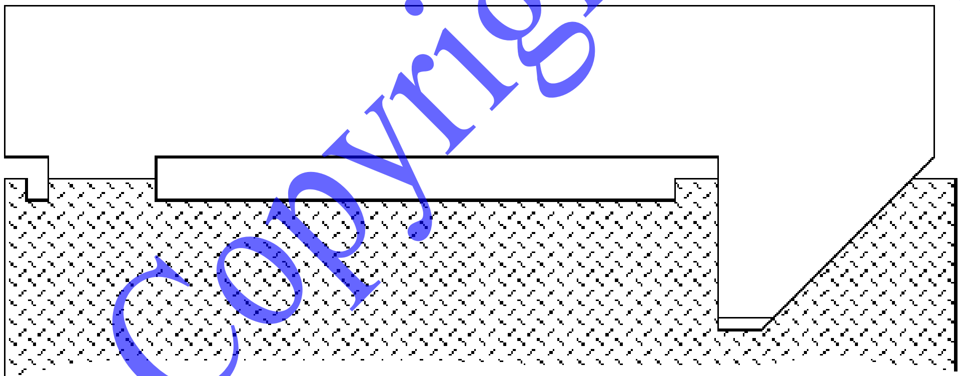
Gravity preloaded bearing configurations

- ❒ **Flat and vee**
  - **Easiest to manufacture.**
  - **When load position changes, center-of-friction changes and yaw loads from actuator cause yaw errors.**



Gravity preloaded bearing configurations

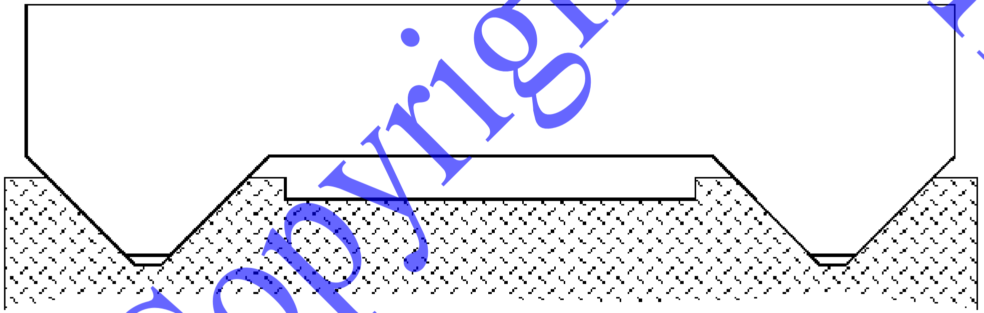
- Flat and half-vee designed to resist side forces (as in a cylindrical grinder)
  - Easy to manufacture.
  - When load position changes, center-of-friction changes and yaw loads from actuator cause yaw errors.
  - When subject to heavy side loads (lathe, cylindrical grinder), carriage will not lift up.



Gravity preloaded bearing configurations

❒ **Double vee**

- Difficult to manufacture.
- Potentially most accurate because of self-checking form and averaging.
- When load position changes, center-of-friction changes only slightly and yaw loads from actuator cause minimal yaw errors.



Size

- Width of bearing pad B

$$B = \frac{W}{pL}$$

Load

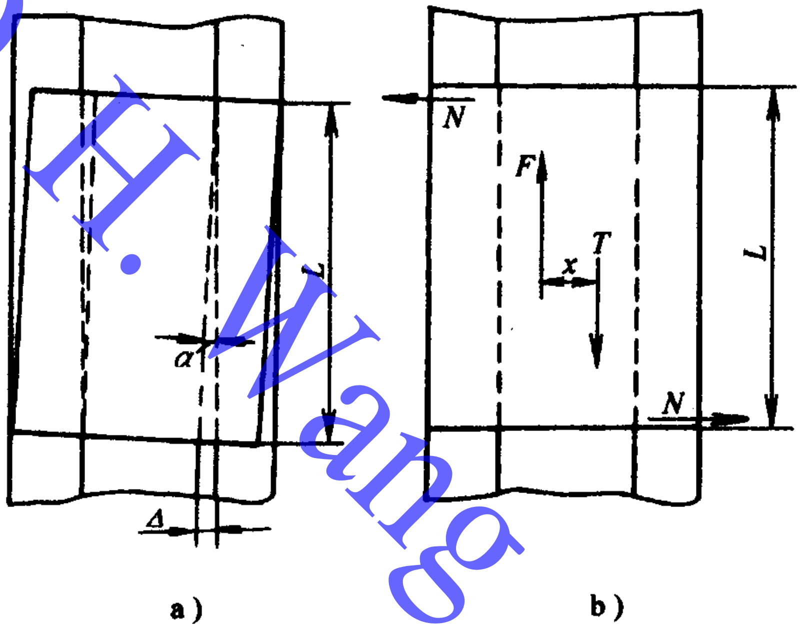
Pressure

Length

- B relates to the capacity of bearing the weigh
- Angle of Vee slides
- Spacing between two slides LA
- Length of carriage

$$\alpha \approx \frac{\Delta}{L}$$

$$N = \frac{Tx}{L} \qquad F = Nf$$



Outline

- ☐ **Characteristics**
- ☐ **Configurations**

Characteristics

- ❒ **Rolling element linear motion bearings are in large part responsible for making automation possible. From a system's perspective, they are one of the most important types of machine elements.**
- ❒ **Speed and acceleration limits**
  - **< 60-120 m/min (2000-4000 ipm) and 1 g.**
  - **At higher speeds, rapidly use up L100 life, and requires oil lubrication.**

Characteristics

❐ **Applied loads**

- Large load capacity is achieved with many elements.
- Remember, load capacity quoted in a catalog is usually for 100 km of travel.
- The load/life relation is cubic:

$$F_{atdesiredtravel} = F_{100\,km\,ratedload}\left(\frac{L_{desin\,km}}{100_{km}}\right)^{-1/3}$$

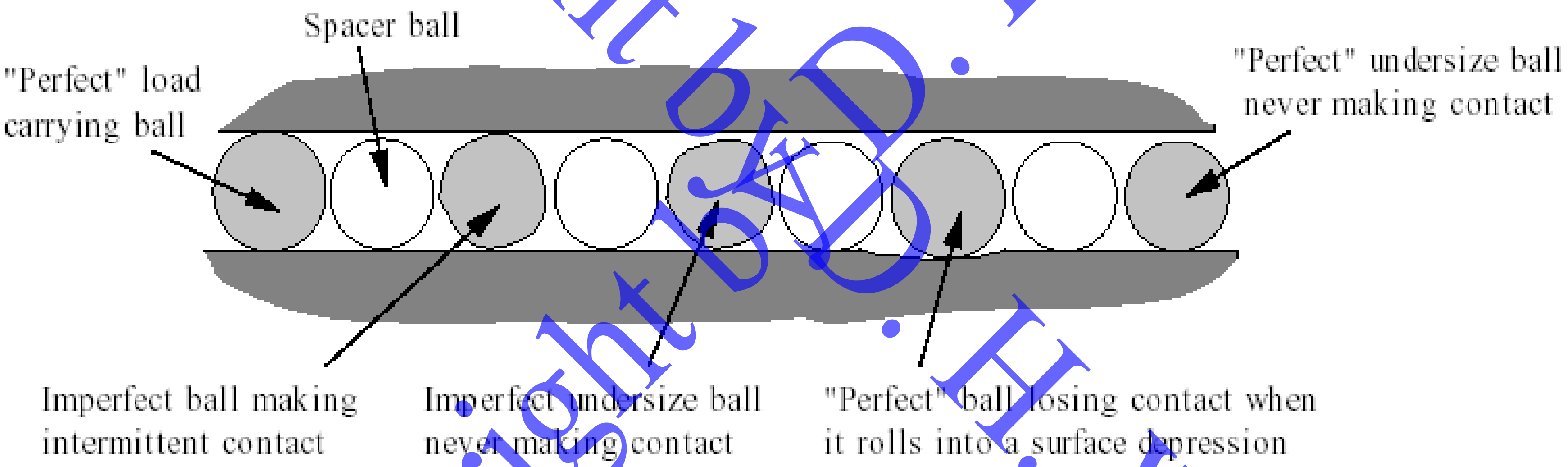
Characteristics

❒ **Accuracy**

- **Axial: 1-5 microns depending on the servo system.**
- **Specially finished systems can have sub-micron accuracy.**
- **Lateral (straightness) : 0.5 - 10 microns depending on the rails and rolling elements.**
- **Rolling elements are not necessarily round and of the same size**
- **Elastic averaging helps to reduce high frequency straightness errors, but they still exist.**
- **Next Slide…**

Characteristics

- ❒ **Accuracy**
  - Elastic averaging helps to reduce high frequency straightness errors, but they still exist.



Characteristics

❒ **Preload**

- **Prevents lost motion upon load reversal.**
- **If an unpreloaded rolling element is separated from the race by a substantial fluid layer:**
  - **The fluid layer directly between the rolling element and the race is incompressible.**
  - **It is driven into the race like a needle, leaving a conical depression.**

❒ **Stiffness**

- **Can be made equal to that of the rest the machine.**
- **Nonlinear (Hertzian), so preload is important.**

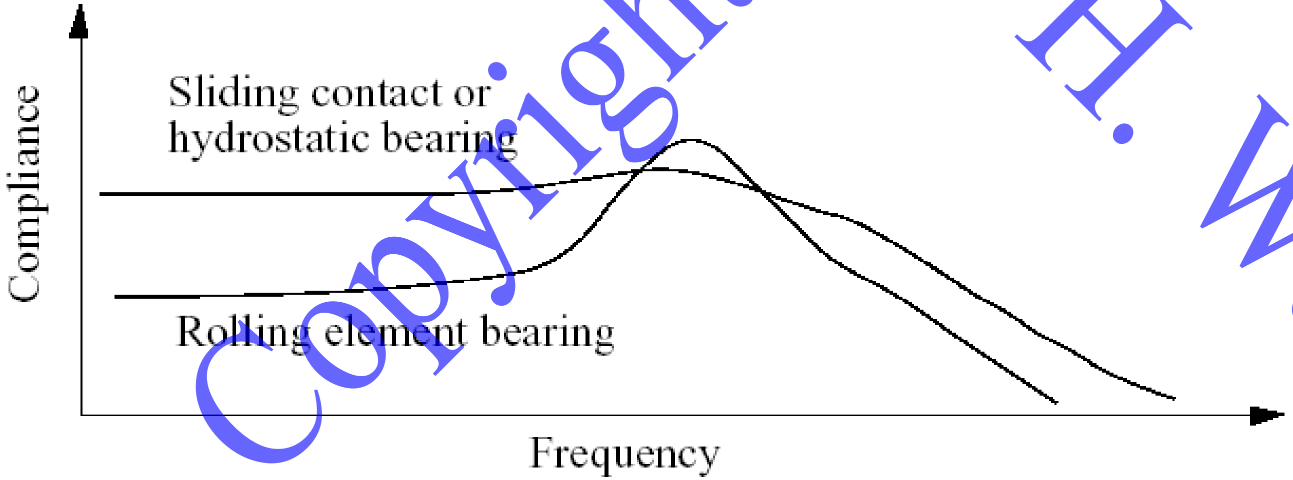
Characteristics

- ❒ **Vibration and shock resistance**
  - Poor to moderate.
  - Significant motion is required periodically to reform a hydrodynamic lubrication layer to prevent fretting.

Characteristics

❒ **Damping**

- Additional damping is obtained from the lubrication layer; however the squeeze film area is very small.
- Along the direction of motion, damping is negligible.
- Non-load carrying sliding contact bearings are sometimes added where damping is very important (e.g., grinders).

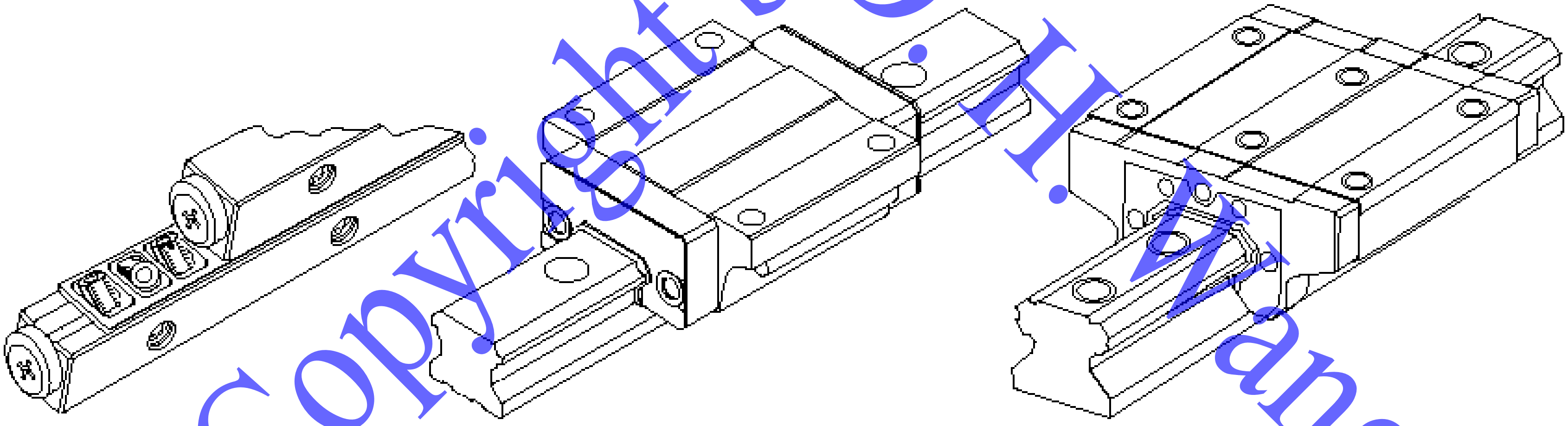


Characteristics

- ❒ Friction
  - Static friction approximately equals dynamic friction at low speeds, so stick slip is often minimized.
  - For heavily loaded tables, static friction is still significantly greater than dynamic friction.
  - Errors will appear at velocity crossovers:
- ❒ Thermal performance
- ❒ Environmental sensitivity
- ❒ Support equipment

Configurations

- ❐ There are a seemingly infinite number of variations on rail geometries and roller handling methods.
- ❐ Typical linear rolling element bearing configurations
  - **Nonrecirculating roller bearing**
  - **Recirculating ball bearing**
  - **Recirculating roller bearing**



General Design Considerations

❒ There are three main types of rolling element linear motion bearings:

- Non-recirculating balls or rollers.
- Recirculating balls.
- Recirculating rollers.

General Design Considerations

❒ **Before choosing a rolling element linear motion bearing, there are several fundamental issues to consider including:**

- **Balls or rollers, which to use?**
- **Shape of the contact surface.**
- **To recirculate or not to recirculate?**
- **Bearing spacing.**
- **Selection criteria.**

Balls or Rollers, Which to Use?

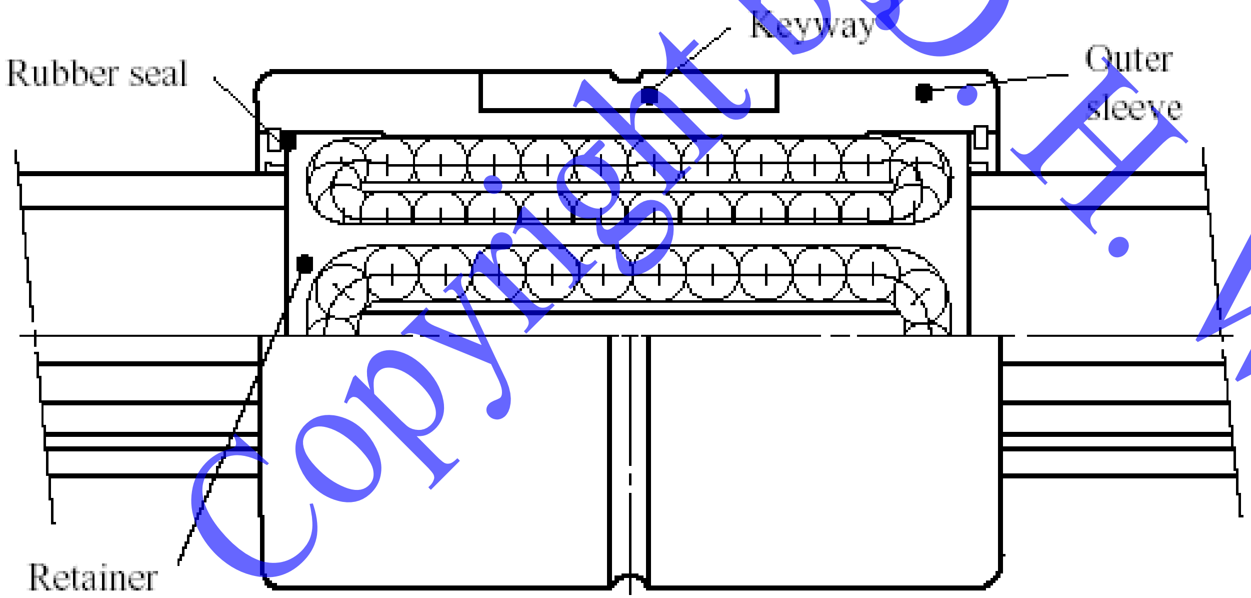
- ❒ **Balls can be made more accurate.**
- ❒ **Balls have no potential to skid sideways.**
- ❒ **Rollers typically have to have a slight barrel shape (or a slightly curved raceway) to avoid edge loading.**
- ❒ **Rollers can have greater load capacity than balls in a circular arch.**
- ❒ **Next slide…**

Balls or Rollers, Which to Use?

- ❐ In the end, all contacts are governed by the Hertz equations, and physics rules over sales talk.
  - Look at the specification sheets.
  - Look at straightness data and rolling element noise spectrums.
- ❐ Build and test a system if necessary.
- ❐ The wise user selects interchangeable components!

To Recirculate or not to Recirculate?

- Recirculating elements allow for "infinite" travel.
- As the elements leave the raceway and enter the raceway, they generate acoustical and straightness noise.
- In most bearings, the elements are not retained, so they can rub on each other causing friction and noise.



To Recirculate or not to Recirculate?

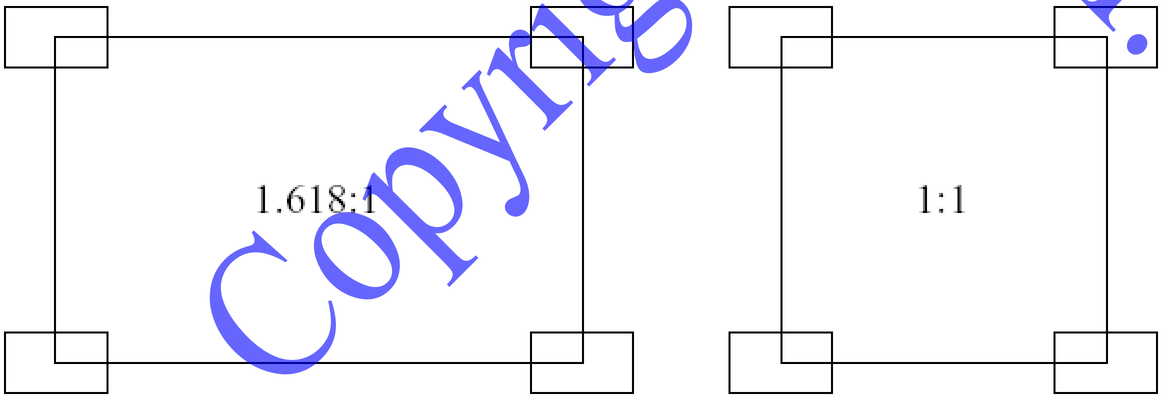
- ❒ Recirculating bearings are often compact and can resist loads and moments from all directions.
- ❒ In general, for short stroke precision applications, it is often best to use non-recirculating bearings.

Bearing Spacing

- ❒ For machine tools, typically the system will be over constrained anyway.
- ❒ The greater the ratio of the longitudinal to latitudinal (length to width) spacing:
  - ✐ The smoother the linear motion will be and the less the chance of walking (yaw error)
- ❒ First try to design the system so the ratio of the longitudinal to latitudinal spacing of bearing elements is about 2:1.
- ❒ Next Slide…

Bearing Spacing

- …
- For the space conscious, the bearing elements can lie on the perimeter of a golden rectangle (ratio about 1.618:1).
- The minimum length to width ratio is 1:1 to minimize yaw error.
- The higher the speed, the higher the length to width ratio should be.

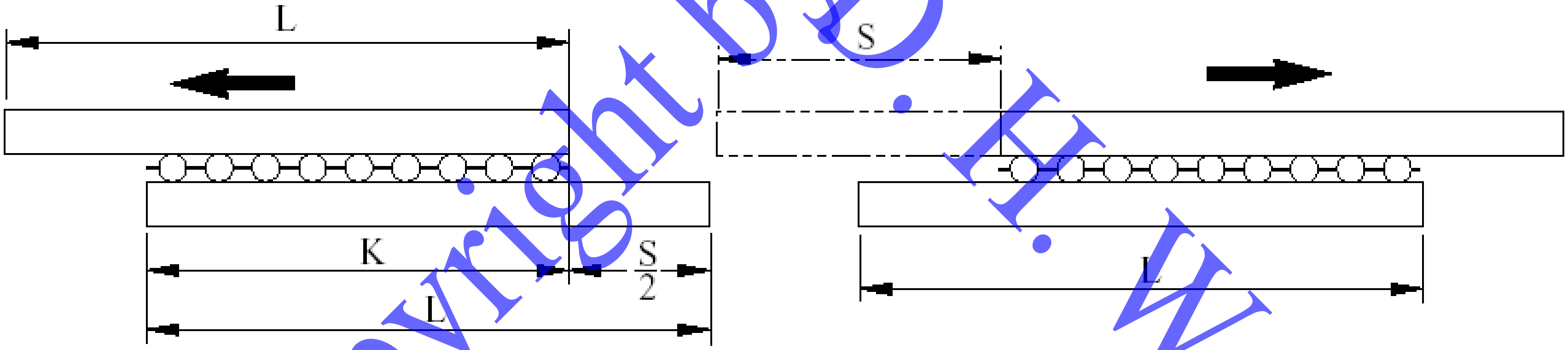


Detailed Design Considerations

- ❒ Performance considerations
- ❒ Running parallelism, repeatability, and resolution.
- ❒ Lateral and moment load support capability.
- ❒ Allowance for thermal growth.
- ❒ Alignment requirements.
- ❒ Preload and frictional properties.

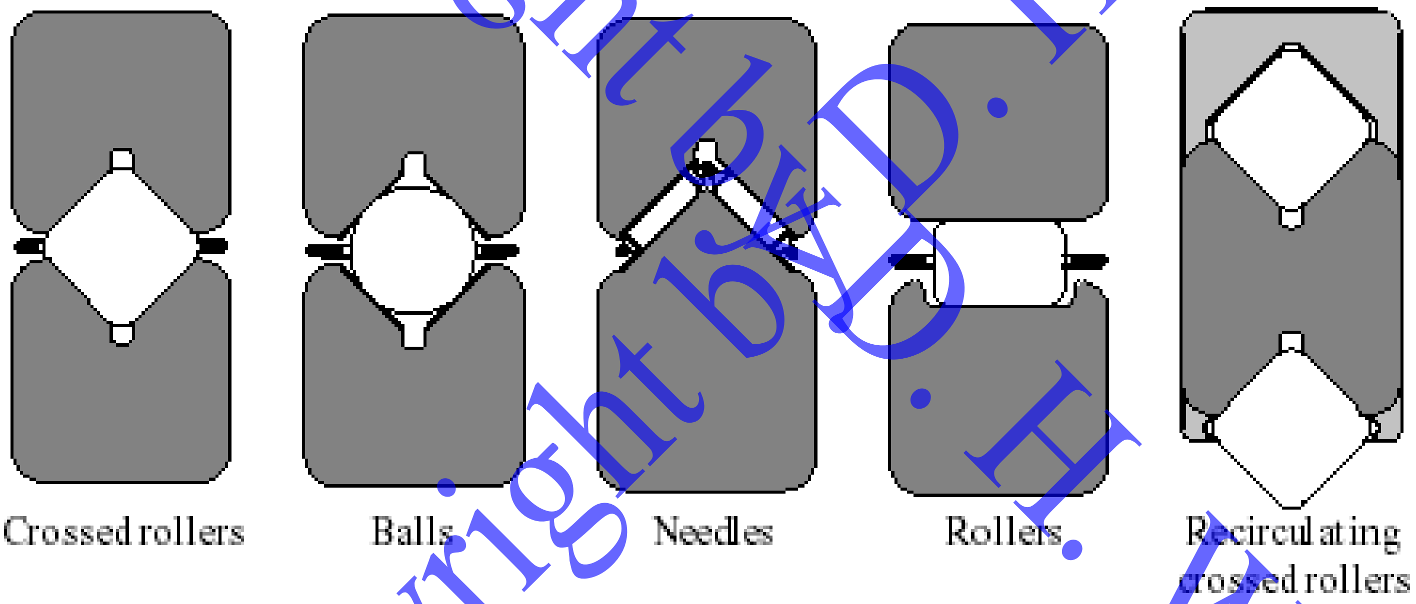
Non-Recirculating

- ❒ **Non-recirculating crossed roller bearings**
  - Quiet, inexpensive, versatile bearing for short travel.
  - Rollers travel half the distance of the moving member:
- ❒ **Variations on the ball or roller in groove theme**



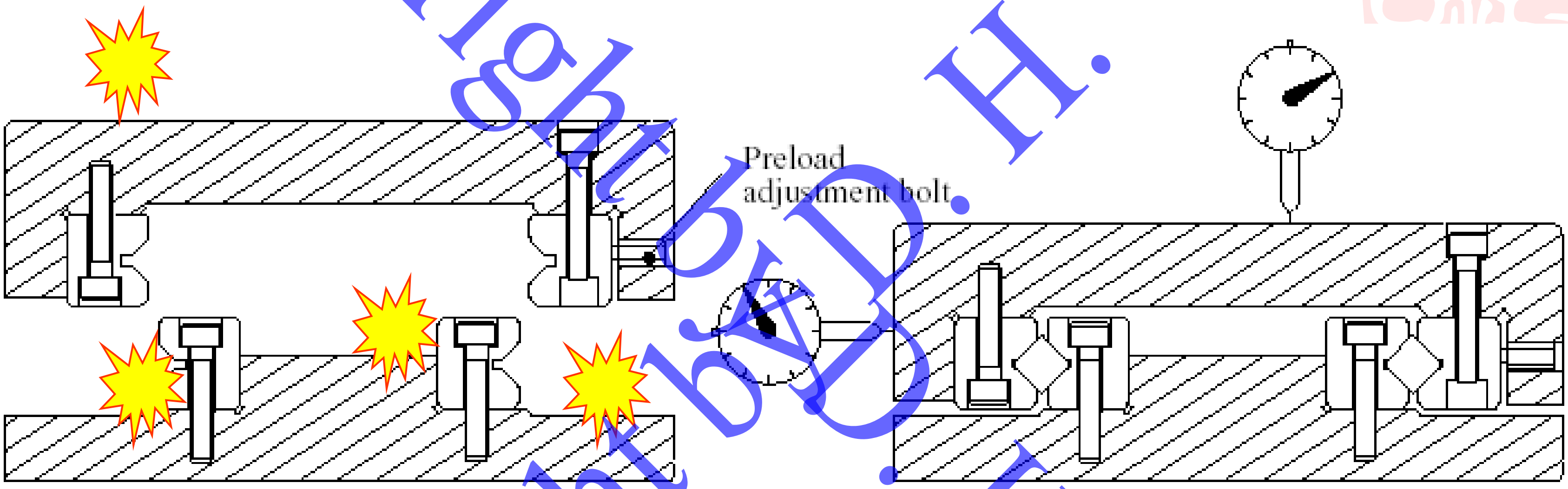
Non-Recirculating

- ❒ Non-recirculating crossed roller bearings
- ❒ Variations on the ball or roller in groove theme



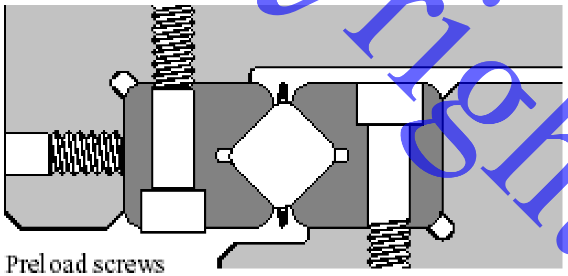
Non-Recirculating

❒ **Typical assembly of crossed roller supported slide:**

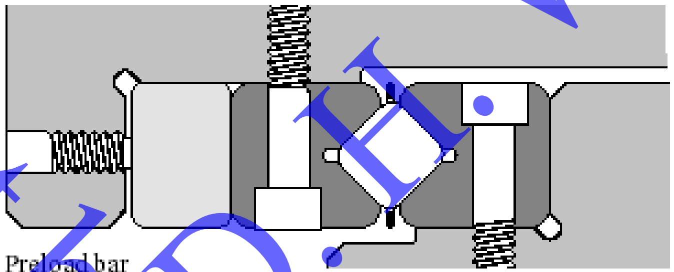


Non-Recirculating

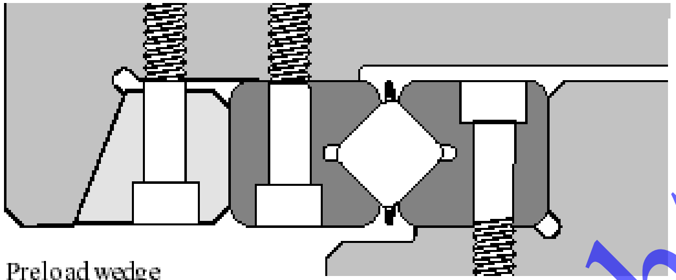
❒ Methods for preloading crossed roller bearings:



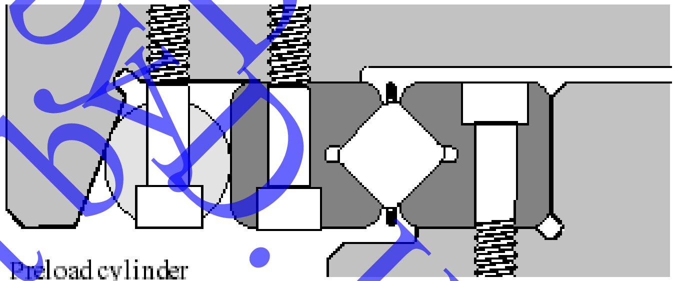
Preload screws



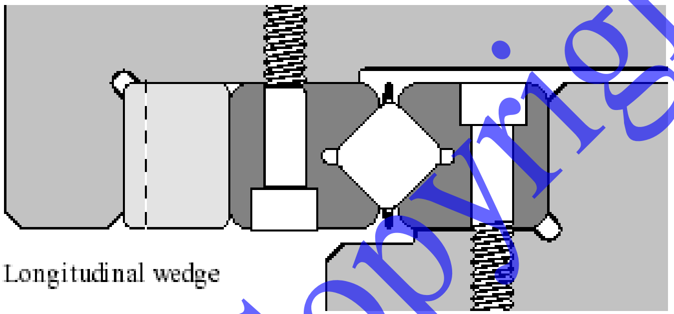
Preload bar



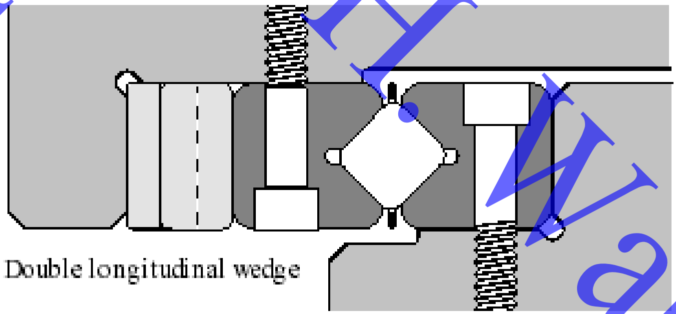
Preload wedge



Preload cylinder



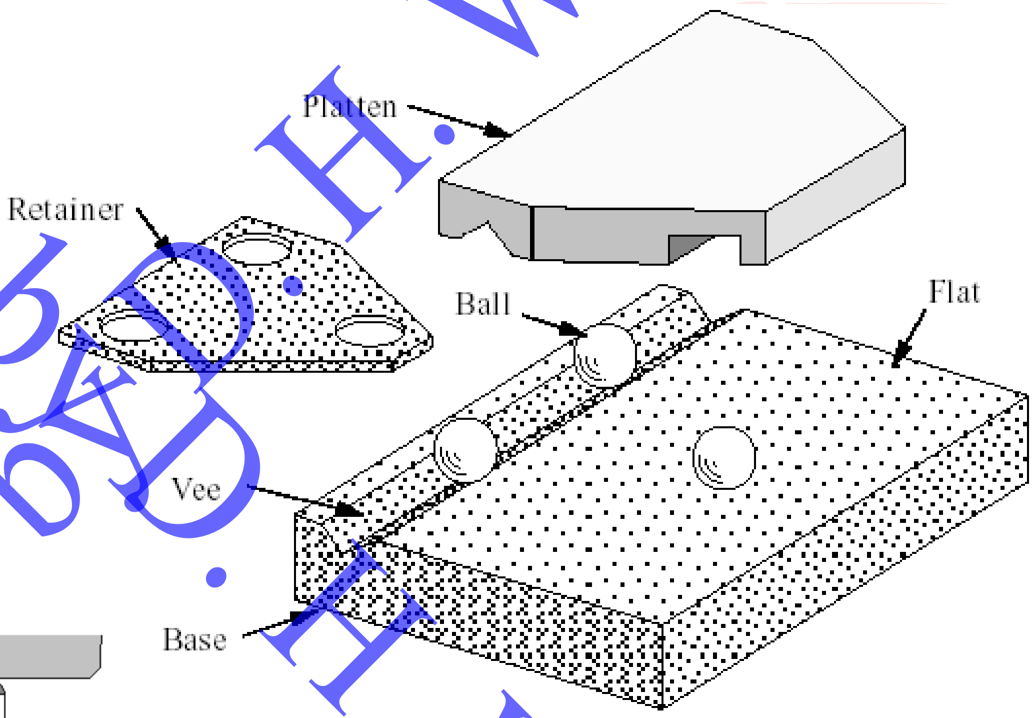
Longitudinal wedge

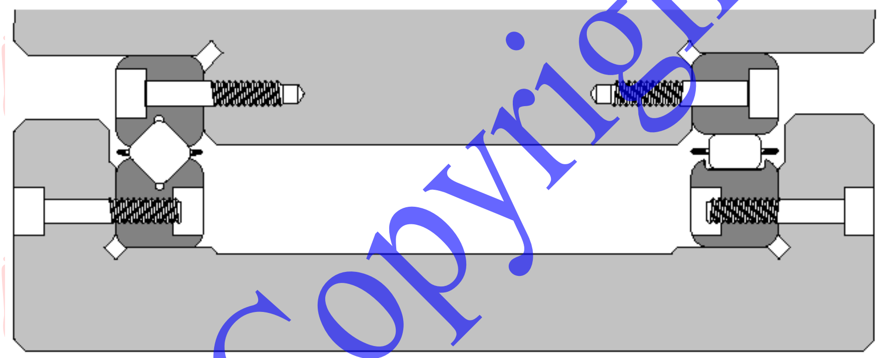


Double longitudinal wedge

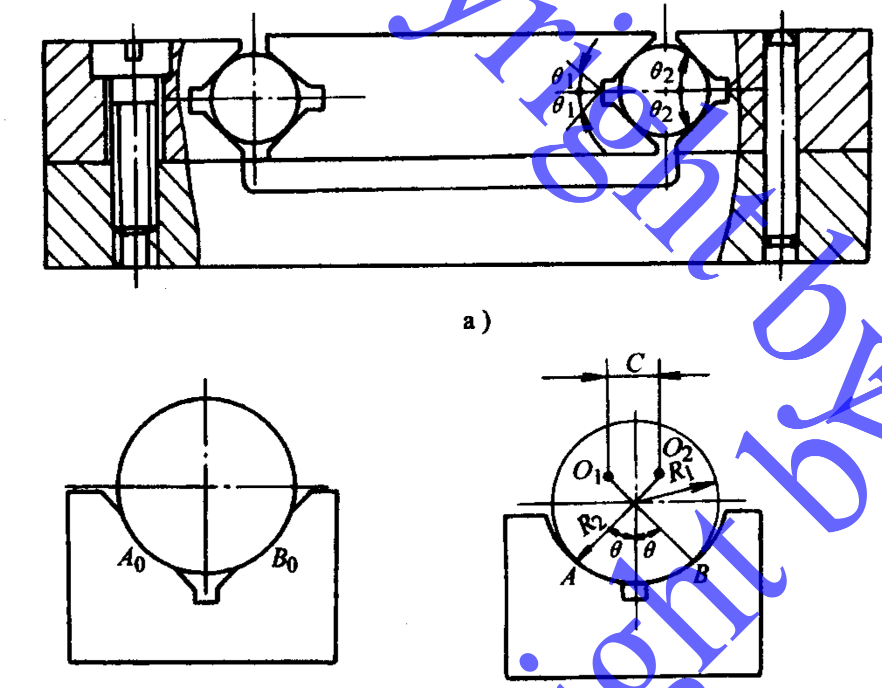
Non-Recirculating

- Kinematic designs are often used:
- Quasi-kinematic arrang rollers on flat rails:





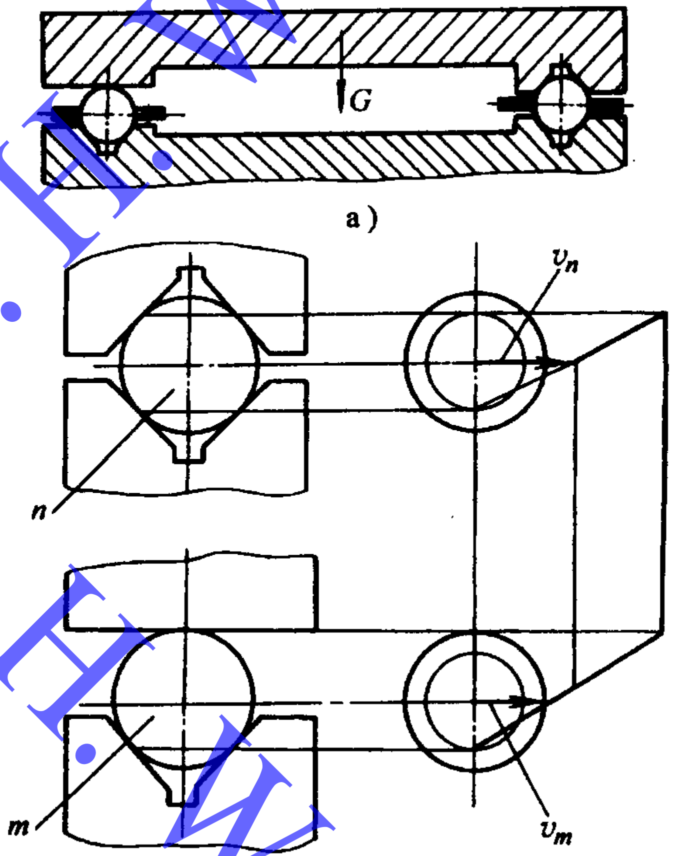
Configurations



a)

b)

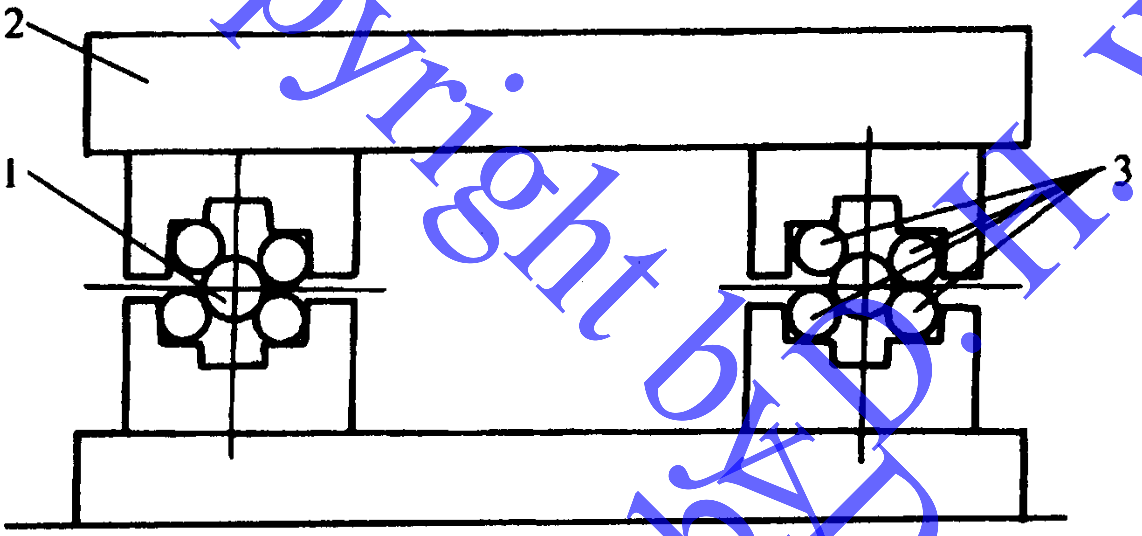
c)



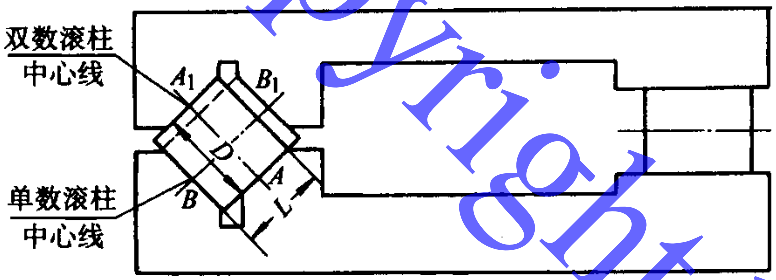
a)

b)

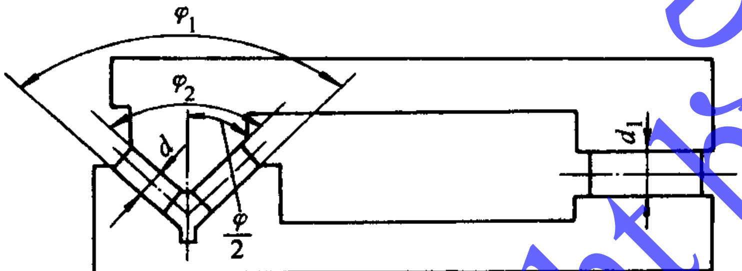
Configurations



Configurations



a)



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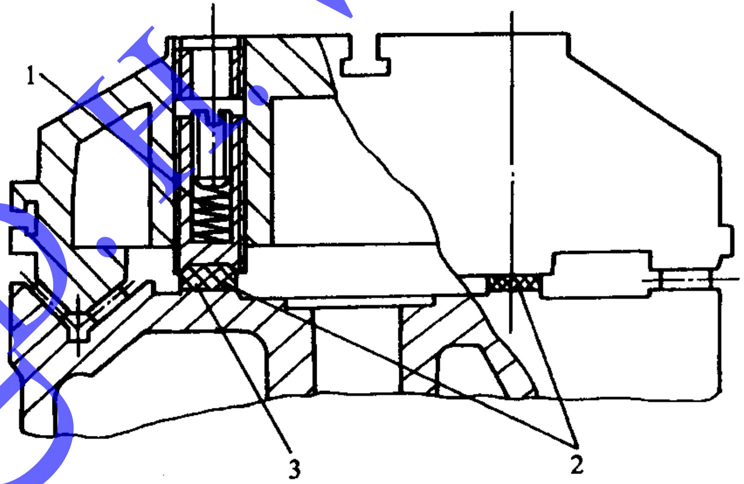


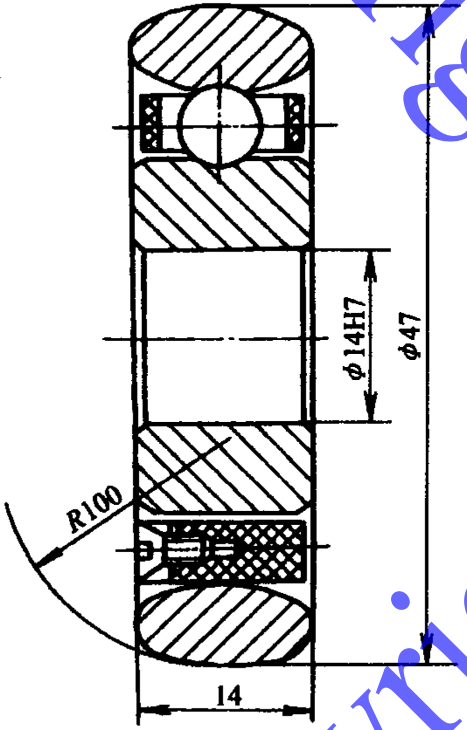
$$d = d_1 \cos\frac{\varphi}{2}$$

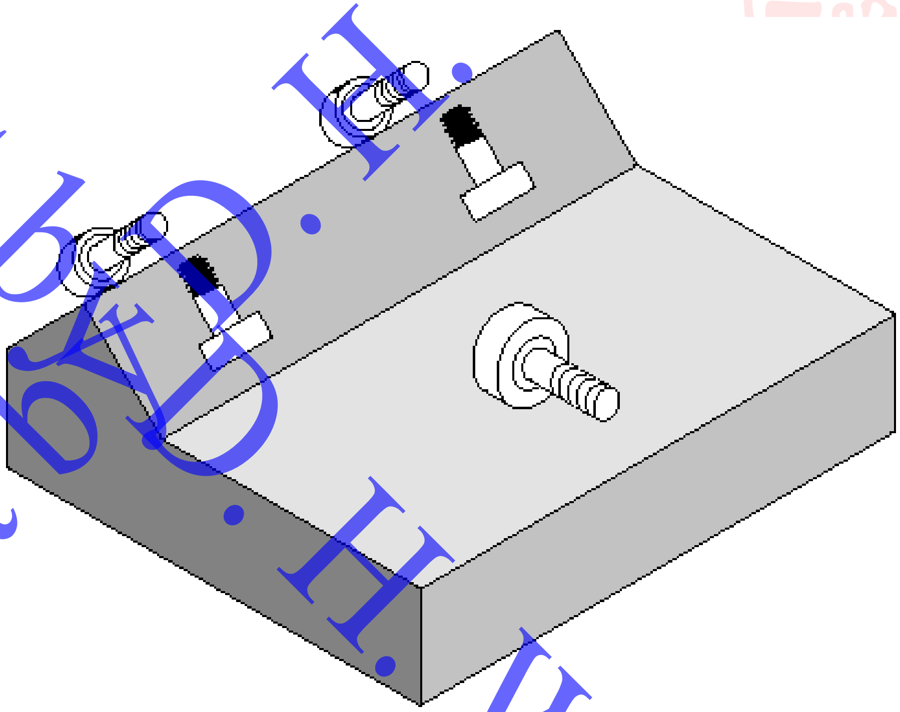
Recirculating

- ❒ Recirculating balls
  - Rotary motion bearings as wheels on rails
  - Recirculating balls on round shafts
  - Recirculating balls on grooved shafts
  - Linear motion guides
- ❒ Recirculating rollers
  - Recirculating rollers on flat rails
  - Recirculating rollers on crowned races
  - Hourglassed-shaped on round ways (X)

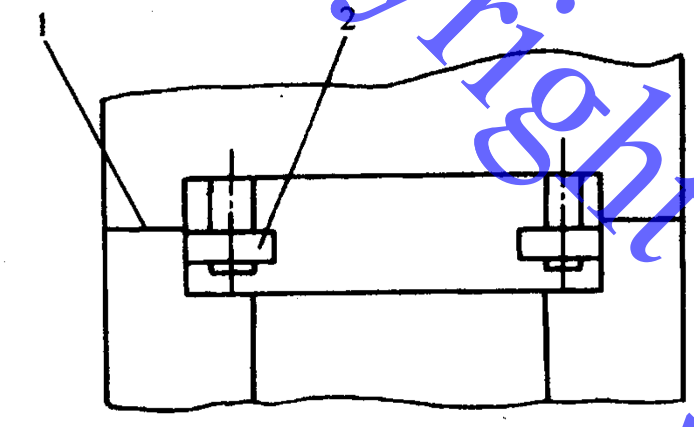
Recirculating

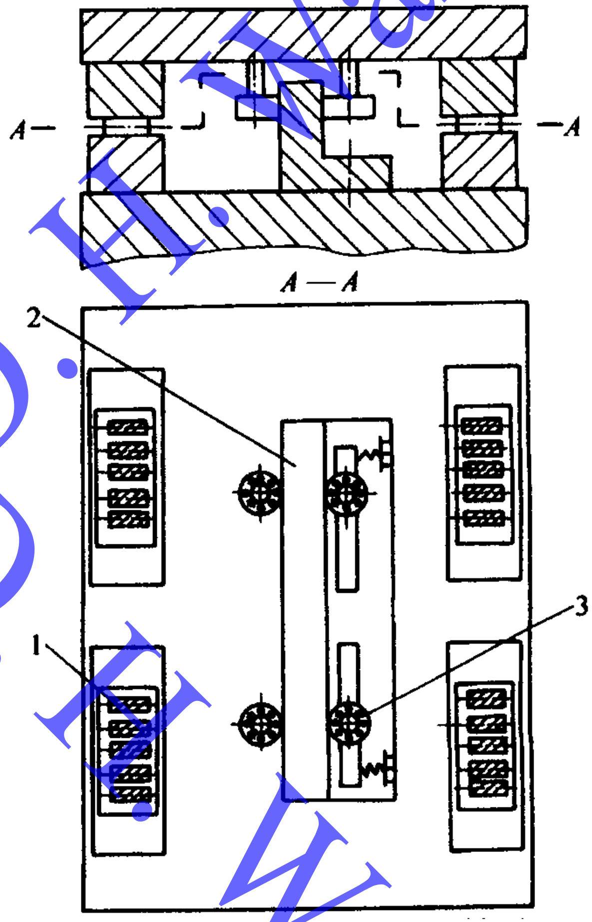
- ❒ **Rotary motion bearings as wheels on rails**





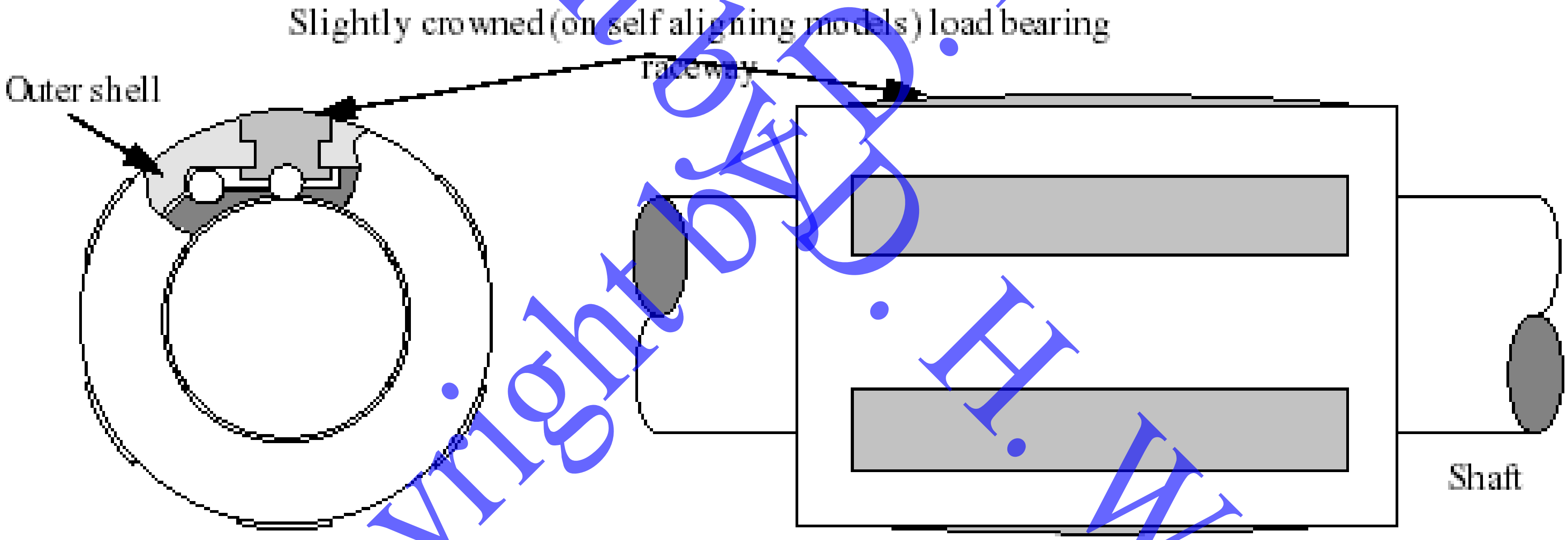
Recirculating





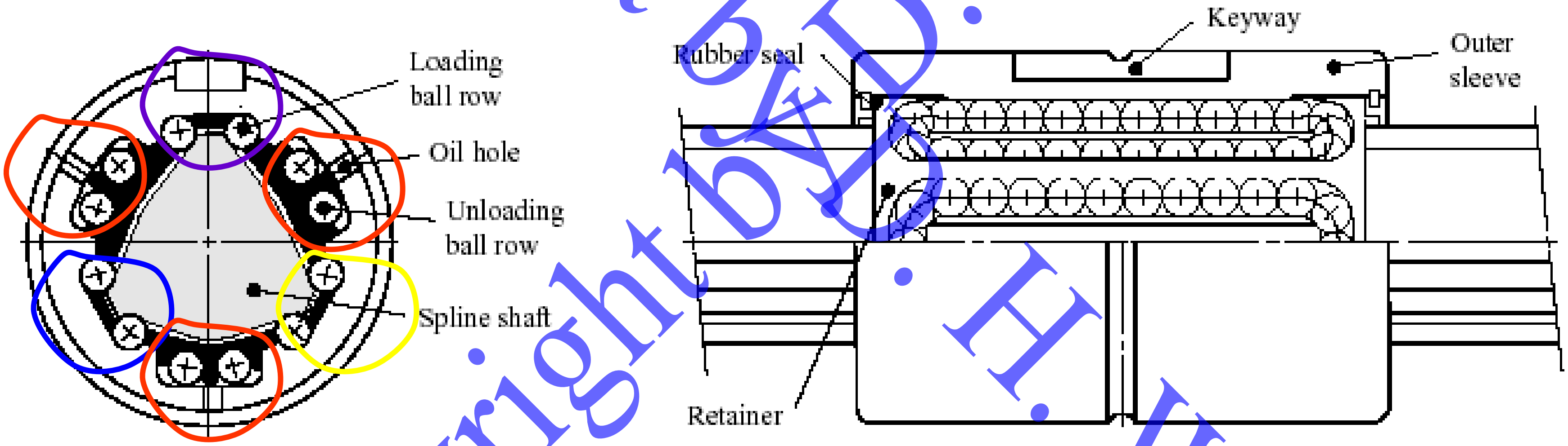
Recirculating

- ❒ **Recirculating balls on round shafts**
  - A linear bearing which incorporates recirculating balls on a round shaft



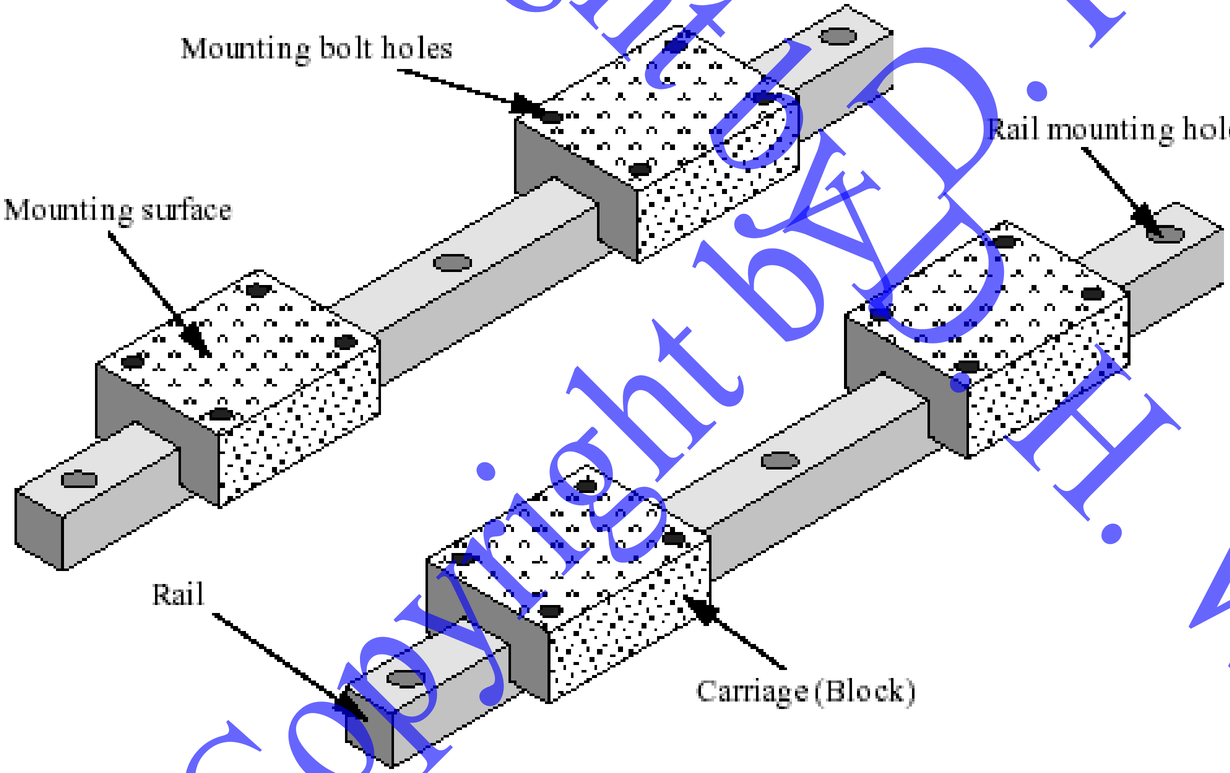
Recirculating

- ❒ **Recirculating balls on grooved shafts**
  - ✐ A linear ball bearing on a shaft with circular arch groove spline.



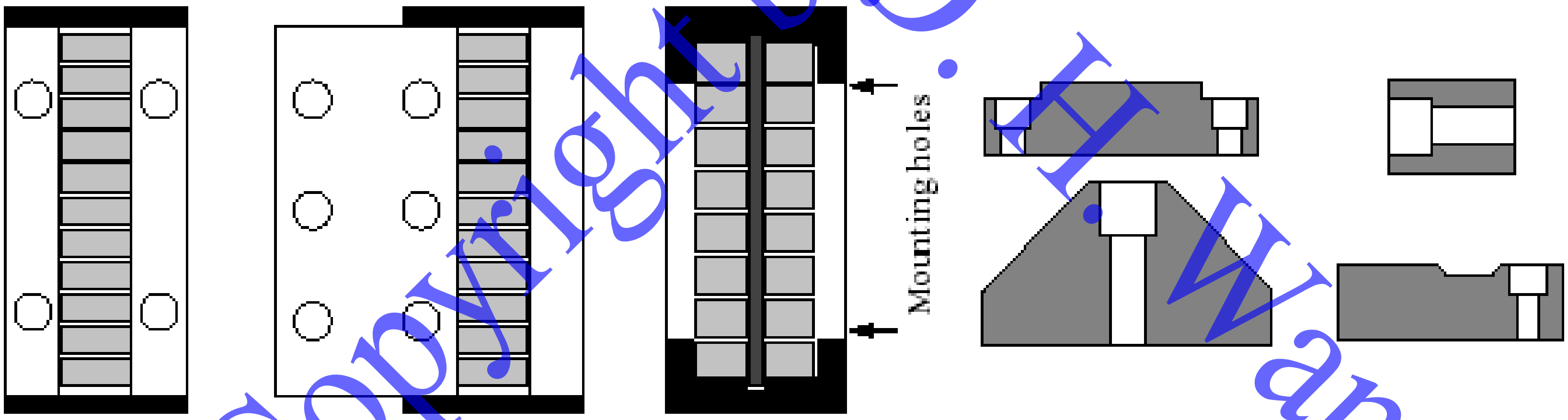
Recirculating

- ❒ **Linear motion guides**
  - **Basic components of a linear motion guide bearing system:**



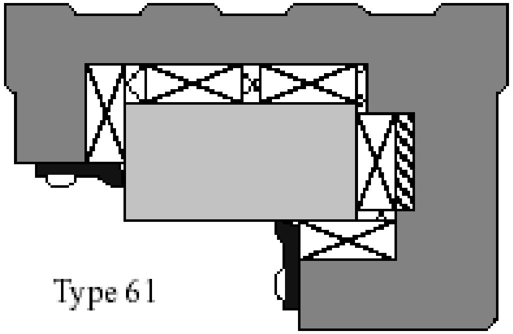
Recirculating

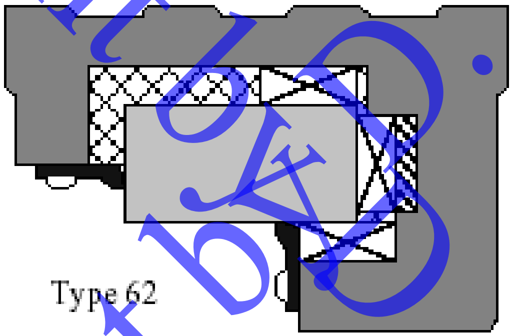
- ❒ **Recirculating rollers on flat rails**
  - Very high load capacity and stiffness, but alignment is critical.
  - Crawler track type recirculating roller bearings for linear motion and some typically available rail types
  - Sometimes called roller packs.

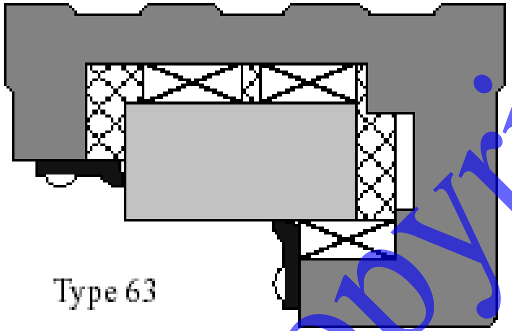


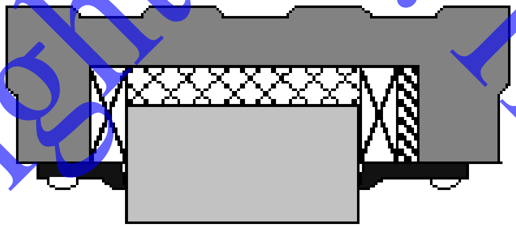
Recirculating

- ❒ **Recirculating rollers on flat rails**
  - Multiple crawler track tread type recirculating roller linear bearings for use on rectangular rails

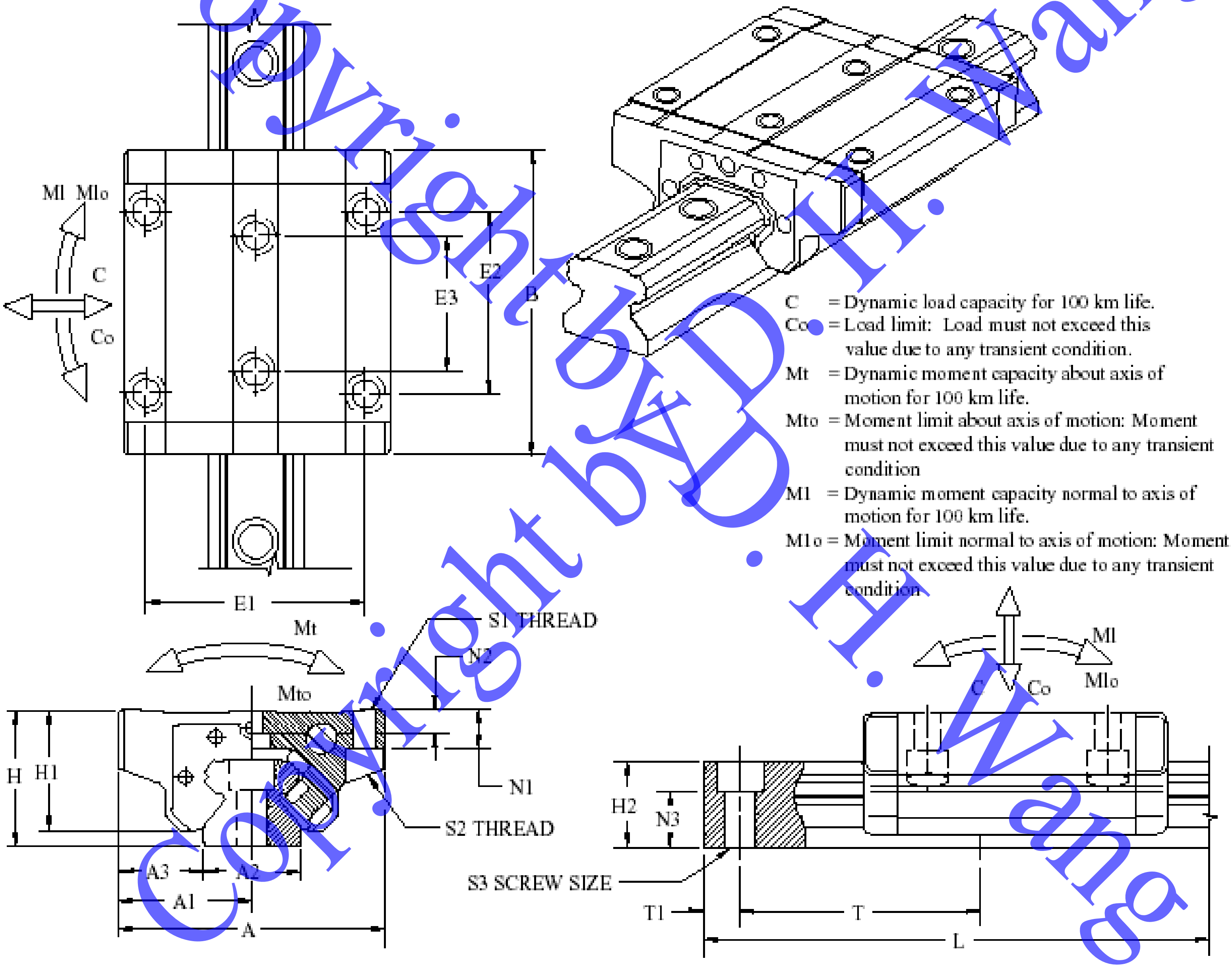

Type 61


Type 62


Type 63


Type 64

Cylindrical Recirculating Rollers on Crowned Races



C = Dynamic load capacity for 100 km life.

Co = Load limit: Load must not exceed this value due to any transient condition.

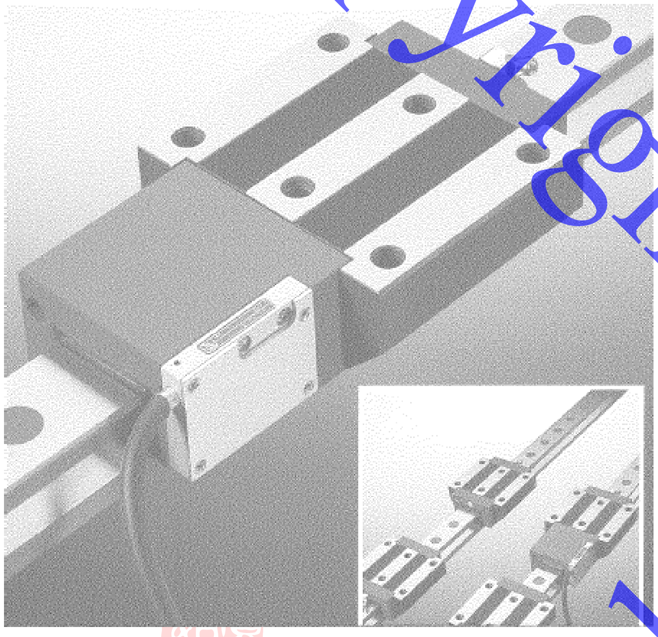
Mt = Dynamic moment capacity about axis of motion for 100 km life.

Mto = Moment limit about axis of motion: Moment must not exceed this value due to any transient condition

Ml = Dynamic moment capacity normal to axis of motion for 100 km life.

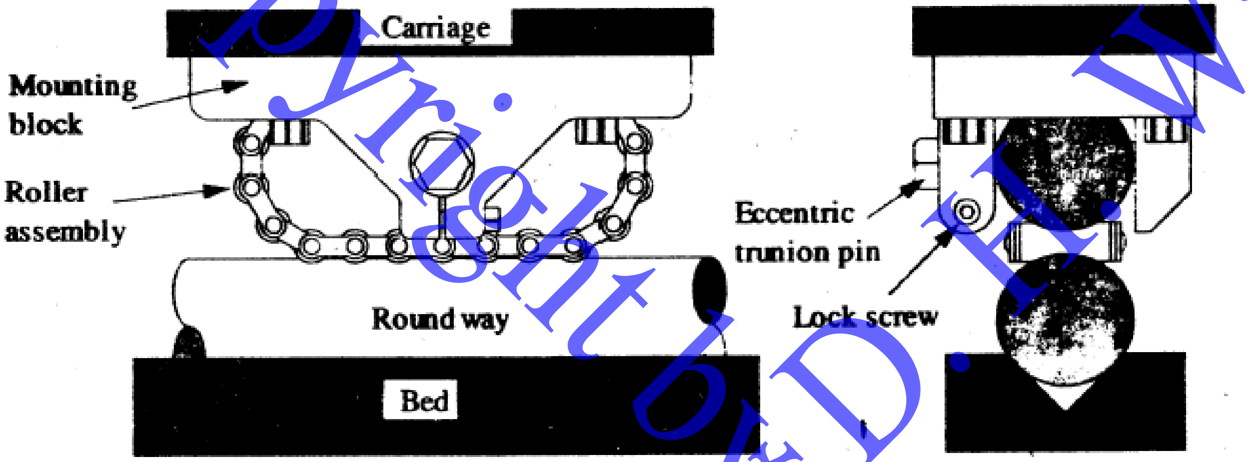
Mlo = Moment limit normal to axis of motion: Moment must not exceed this value due to any transient condition

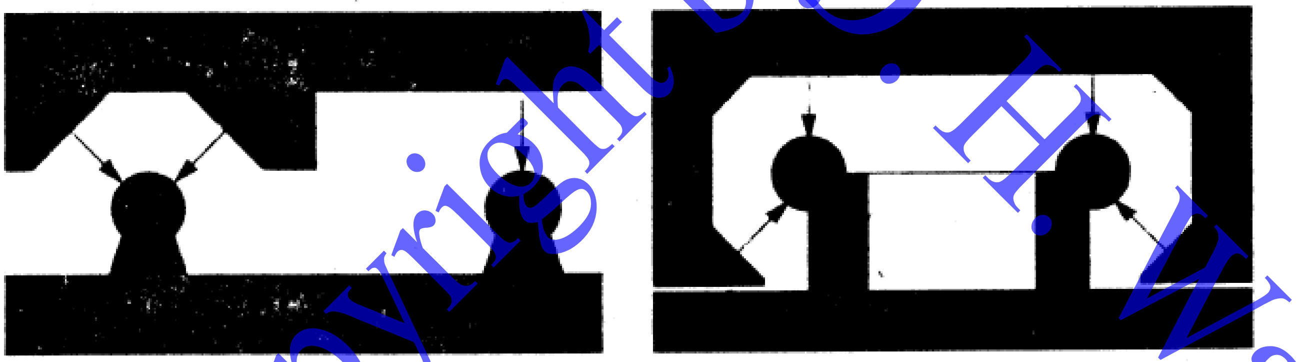
Integrated linear bearing and sensor



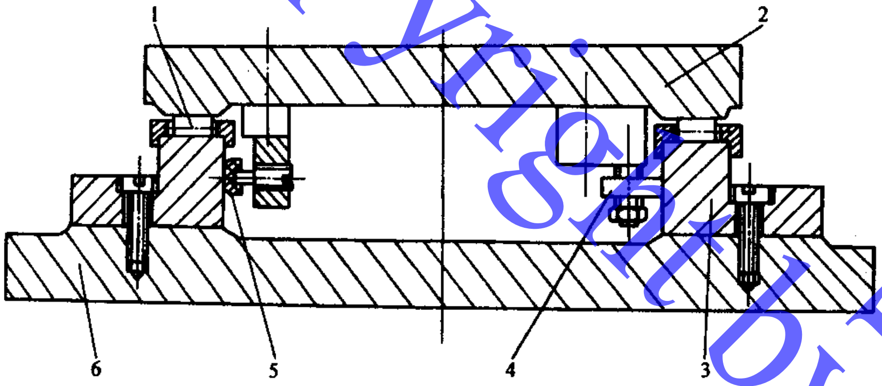
- ❒ Monorail bearing developed by Schneeburger to eliminate assembly & alignment issues associated with linear encoders.
- ❒ Magnetic scale is attached to the rail, and the read-head to the carriage.
- ❒ Recirculating rollers on flat surfaces on profile rails.
- ❒ Line contact generally gives this type of bearing the highest load capacity, stiffness, and damping.

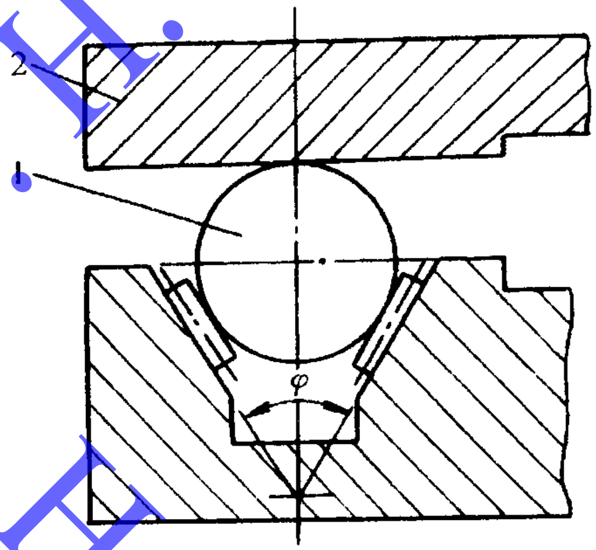
Hourglassed-shaped on round ways

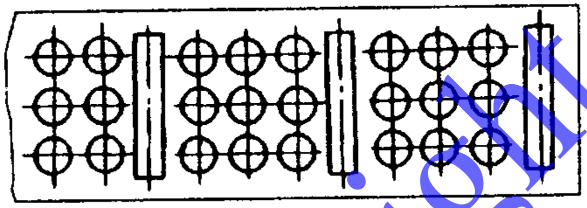




Combinations







E-mail: dhwang@cqu.edu.cn
URL: http://www.pilab.coe.cqu.edu.cn/

Size

- **Length of carriage Ld**

$$L_B = L_d + \frac{l}{2} \qquad L_d = L_B - \frac{l}{2}$$

- **Size and number of rollers and/or balls**
  - **Size of rolling elements**

Load capacity $\propto f\left(z, d^2\right)$

  - **Number of rolling elements**

$$z_{柱} \leqslant \frac{W}{4l} \qquad z_{珠} \leqslant \frac{W}{9.5\sqrt{d}}$$

- **Strength and stiffness**

Strength and Stiffness

- ☐ Length of carriage Ld
- ☐ Size and number of rollers and/or balls
- ☐ Strength and stiffness

$$p_{max} < p_a \qquad p_{max} = p_{smax} t B$$

Contact strength

Roller bearings $p_a = \sigma_k d l \xi$

Ball bearings $p_a = \sigma_k d^2 \xi$

Contact stiffness

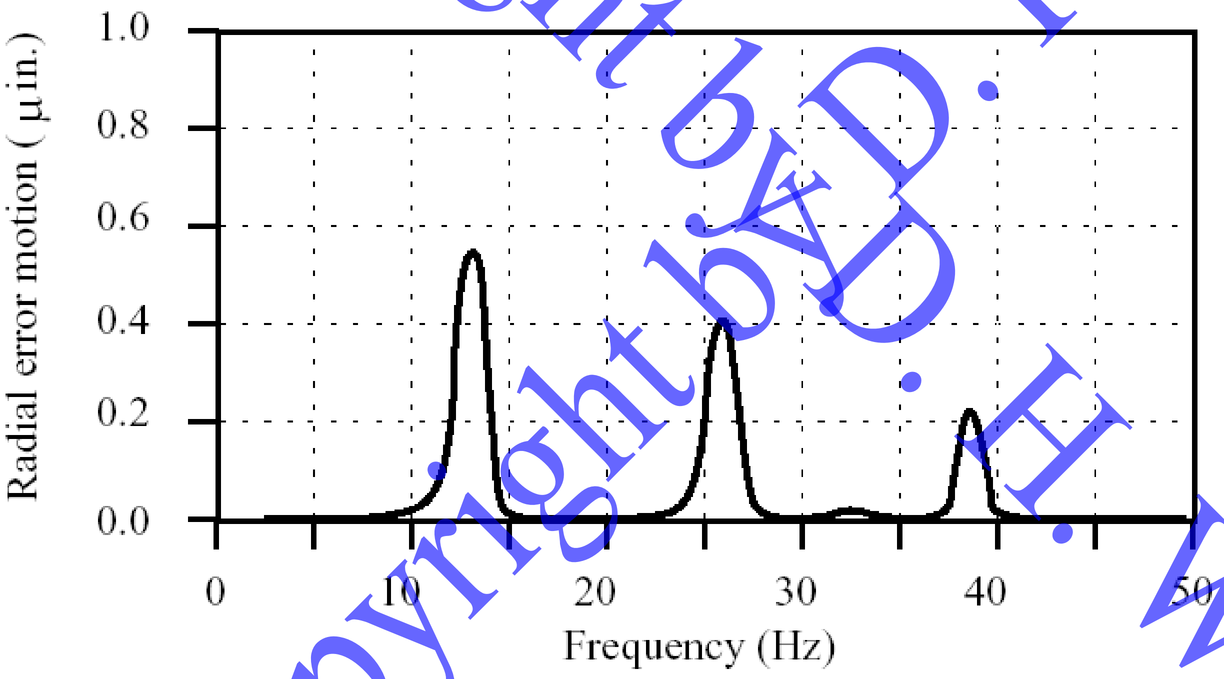
Roller bearings $\delta = c_1 P_c$

Ball bearings $\delta = c_2 q_c$

Outline

General Characteristics

- ❒ Lack of mechanical contact between elements causes error motions to be small, and harmonics quickly die out

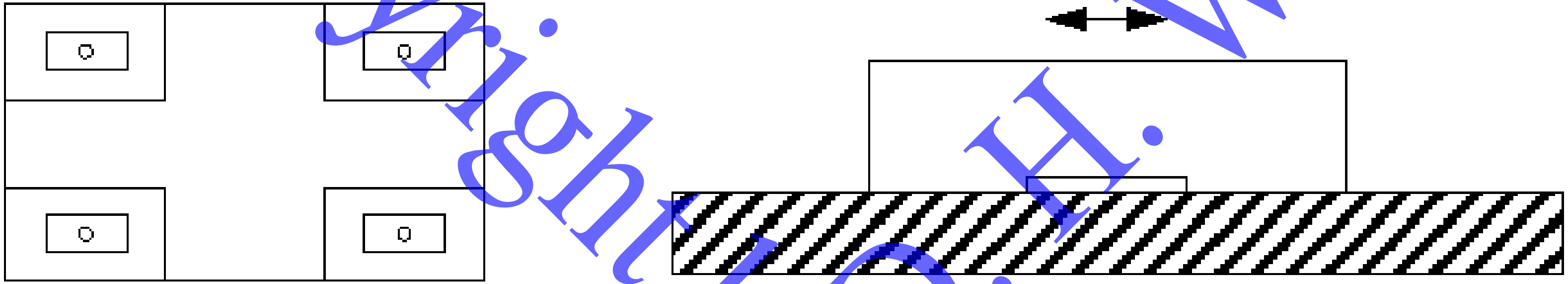


General Characteristics

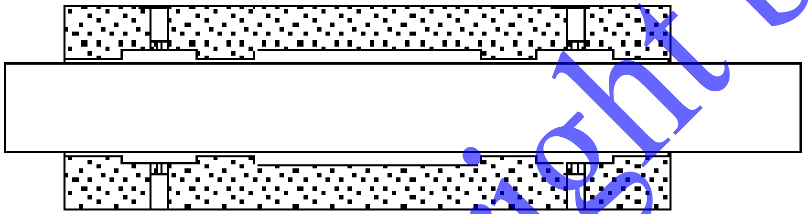
❒ Hydrostatic and aerostatic bearings use an external pump to supply pressureized fluid to the bearing:

- Metered flow to each side of the bearing creates a pressure differential proportional to the displacement.
- Load capacity and stiffness can be very high.
- They require the expense of a clean pressure supply system.

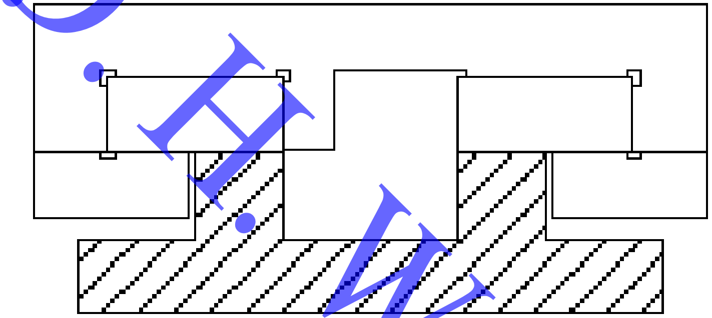
Configuration



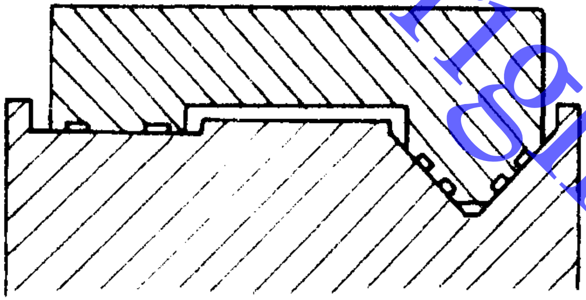
Single pad hydrostatic bearings used to support a two degree of freedom platen



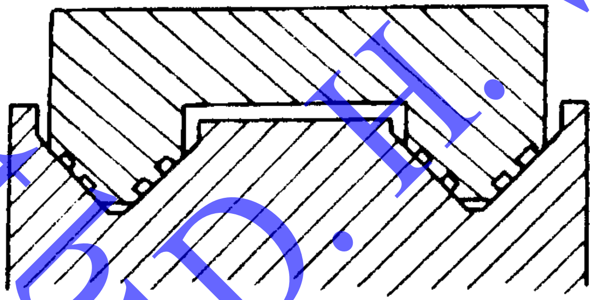
Opposed pad bearings used to support a machine tool carriage



Configuration



a)



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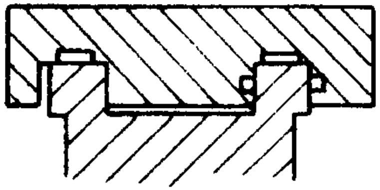

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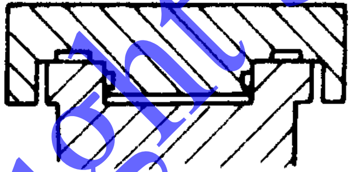

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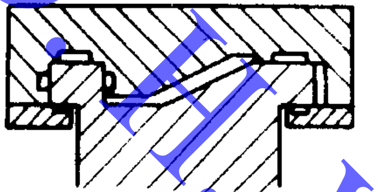
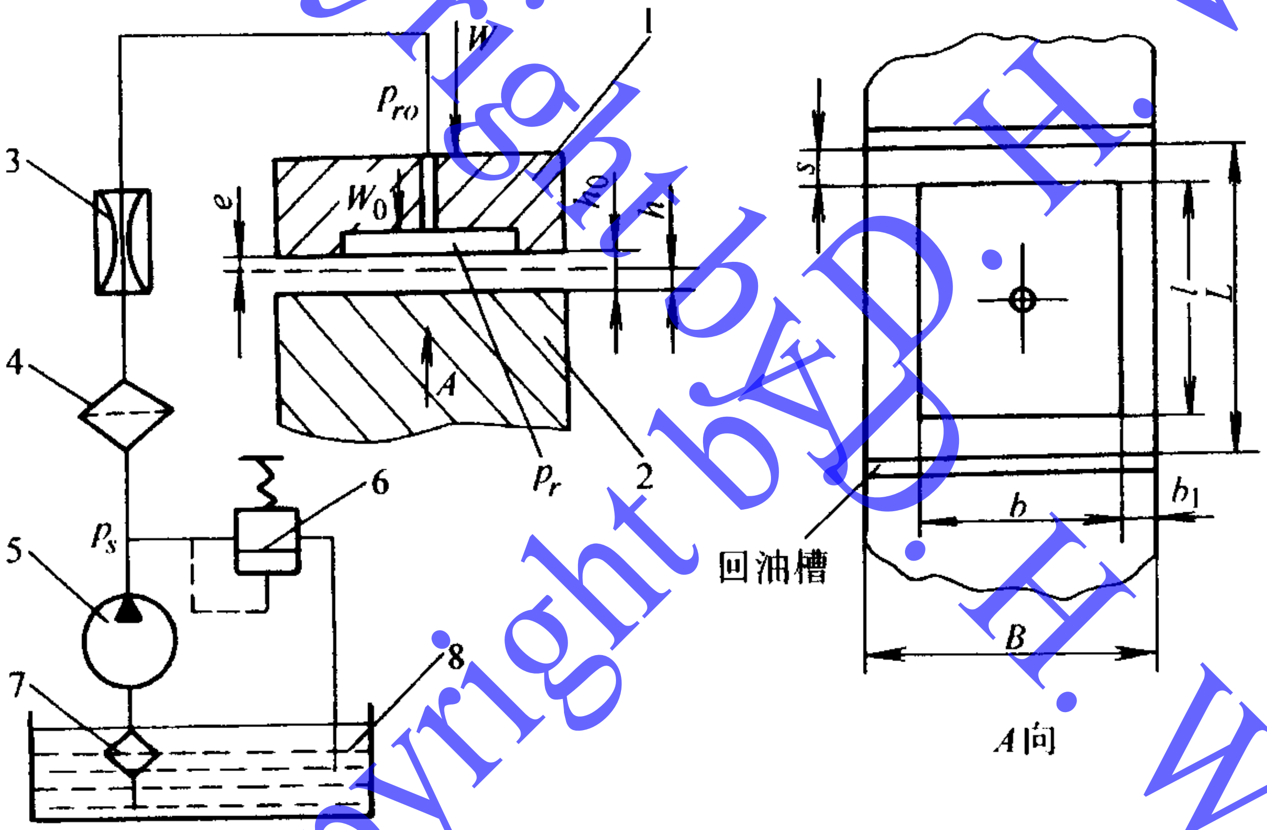


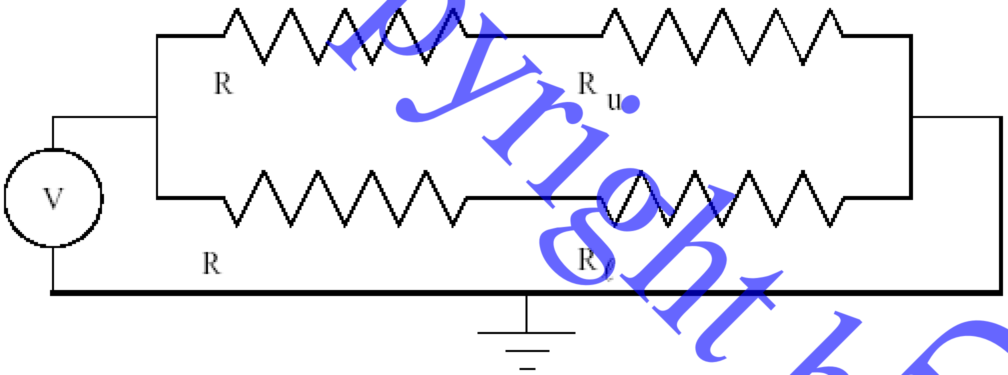
e)

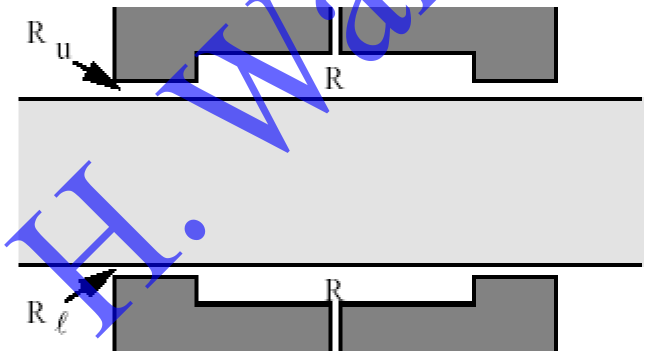
Working Principle

❒ Pump



Theory of Operation for Plane Opposed Bearings with Fixed Compensation

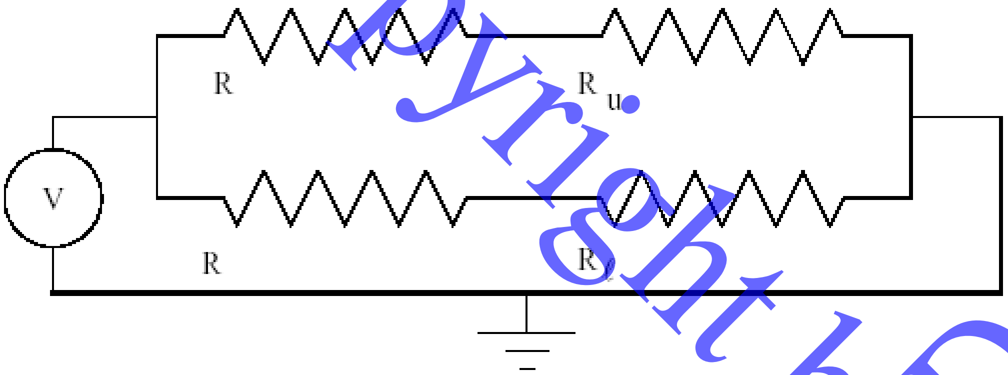


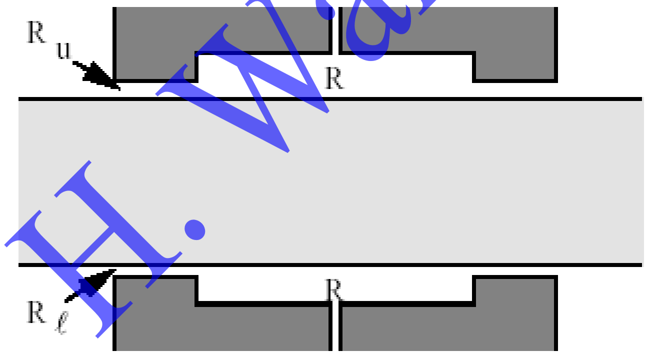


- Fluid flow into the bearing is regulated (R) by a resistance.
  - When a force applied to the bearing, the fluid flow resistance changes.
  - A load-balancing pressure differential is developed
- The difference in pressure between the upper and lower pads of the bearing is:

$$\Delta P = P_u - P_l = P_s \left(\frac{R_u}{R + R_u} - \frac{R_l}{R + R_l} \right)$$

Theory of Operation for Plane Opposed Bearings with Fixed Compensation





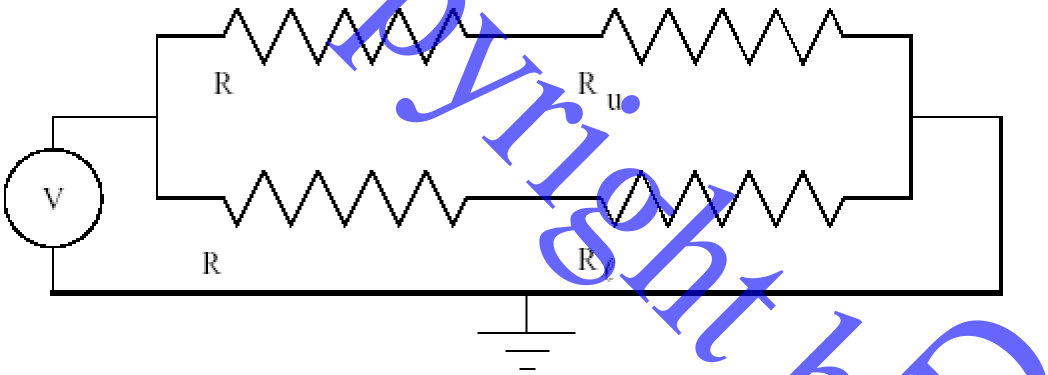
- For a nominal gap h and small excursions δ of the structure:

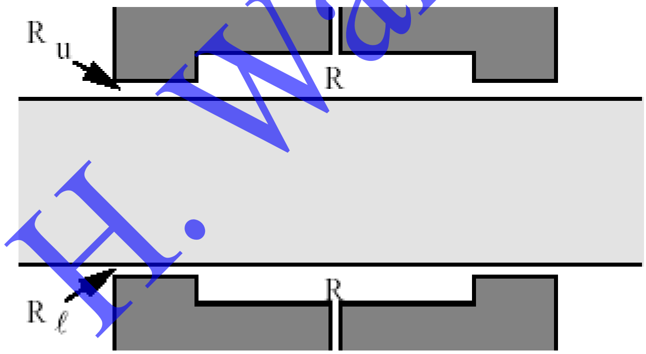
$$R_u = \frac{\gamma}{(h - \delta)^3} \qquad R_l = \frac{\gamma}{(h + \delta)^3}$$

- The difference in pressure across the bearing is:

$$\Delta P = P_s \gamma \left[\frac{1}{R(h - \delta)^3 + \gamma} - \frac{1}{R(h + \delta)^3 + \gamma} \right]$$

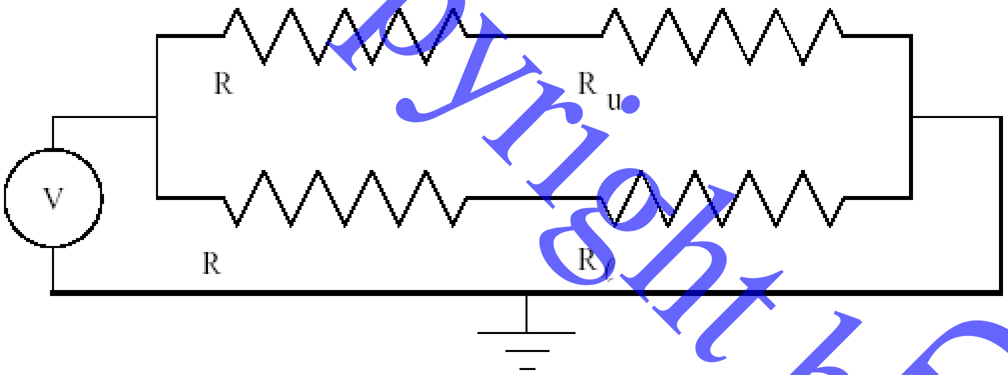
Theory of Operation for Plane Opposed Bearings with Fixed Compensation

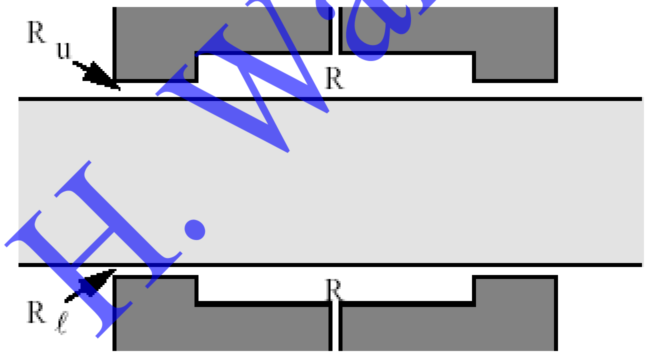




- If the inlet flow resistance R was zero, the bearing could support no load.
- If the inlet flow resistance was infinite, the bearing could support no load.
- There must be some ideal inlet resistance (compensation) between these two extremes.

Theory of Operation for Plane Opposed Bearings with Fixed Compensation





- ❐ Taking the partial derivative of the pressure difference with respect to the inlet flow resistance; Ignoring all terms with δ2 and higher terms:

$$\frac{\partial \Delta P}{\partial R} = P_s \gamma h^2 \left[\frac{-(h-3\delta)}{\left[Rh^2(h-3\delta)+\gamma\right]^2} - \frac{(h+3\delta)}{\left[Rh^2(h+3\delta)+\gamma\right]^2} \right]$$

- ❐ The "optimal" inlet flow resistance to maximize load capacity is

$$R = \frac{\gamma}{h^3}$$

Theory of Operation for Plane Opposed Bearings with Fixed Compensation

- **If the displacement of the bearing is assumed to be a portion of the nominal gap, δ = αh:**

$$\Delta P = P_s\left(\frac{1}{(1-\alpha)^3+1} - \frac{1}{(1+\alpha)^3+1}\right)$$

- **Linearizing about α = 0:**

$$\Delta P \approx P_u - P_l \approx \frac{P_s}{2-3\alpha} - \frac{P_s}{2+3\alpha} \approx \frac{3P_s}{2}\alpha$$

- **For an opposed pad bearing with supply pressure Ps and inlet restrictor resistance R, the total flow is just Q = Ps/R.**

Theory of Operation for Plane Opposed Bearings with Fixed Compensation

- ❒ **Stiffness is the change in load for a given change in bearing gap (Aeffective ∂ΔP/∂δ) where Aeffective is the effective bearing**

$$K = A_{effective}\frac{\partial \Delta P}{\partial \delta} = 3P_s\, A_{effective} h^3 \left[\frac{(h - \delta)^2}{\left[(h - \delta)^3 + h^3\right]^2} + \frac{(h + \delta)^2}{\left[(h + \delta)^3 + h^3\right]^2} \right]$$

- ❒ **At maximum load capacity, the bearing stiffness is:**

$$K \approx \frac{3P_s\, A_{effective}}{2\,h} \approx \frac{F_{max}}{h}$$

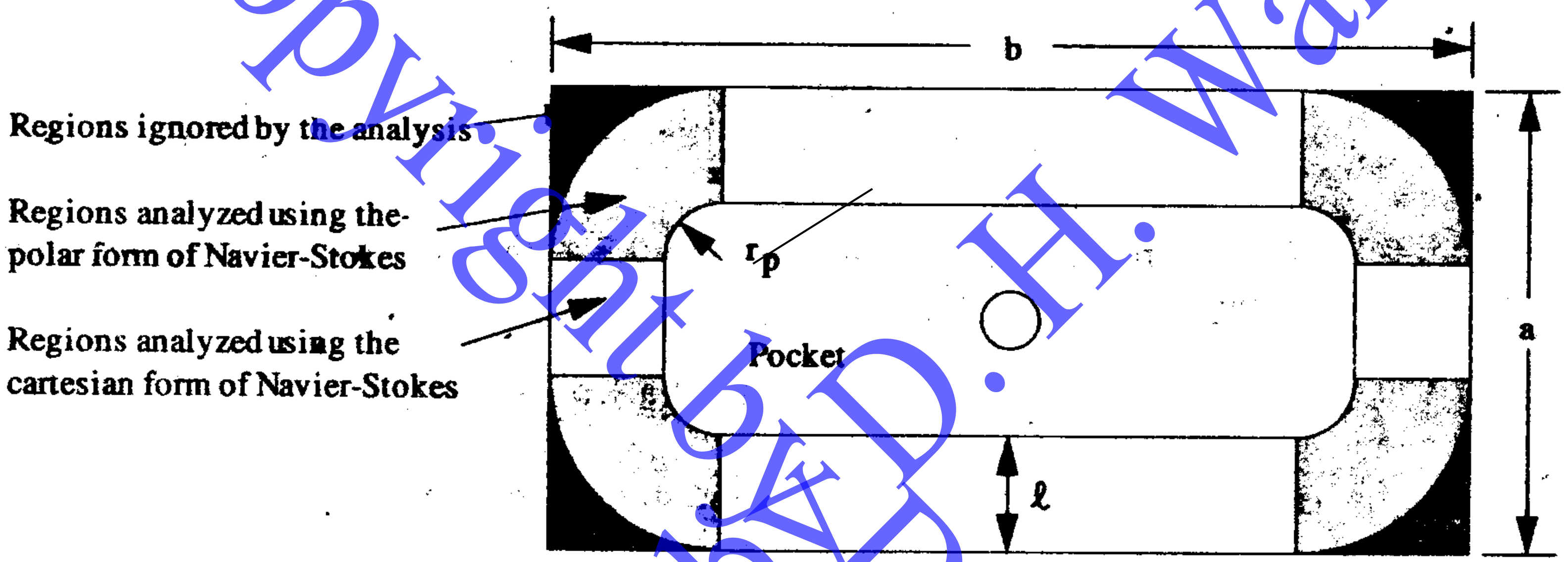
Theory of Operation for Plane Opposed Bearings with Fixed Compensation

- **If P = 2MPa (20 atm), a=b=0.05 m, Aeffective=0.001250 m2 and h=10 μm , then K=375 N/μm which is a very stiff bearing.**
- **The load the bearing can support is F = Kδ, where δ = αh:**

$$F \approx \frac{3P_S\,A_{effective}\alpha}{2} \approx \frac{P_S\,A_{total}}{2}$$

- **With α = 0.5 and a correction factor from Figure 9.2.3 of 0.88, the bearing load capacity is 1650 N (371 lbf).**

Bearing Effective Area



- Pad flow resistance
  - The fluid resistance of the straight sections of the flat pad bearings
  - The fluid resistance of the rounded corner regions
  - The total resistance of the rectangular flat pad pocketed bearing

Bearing Effective Area

- ❒ The fluid resistance of the straight sections of the flat pad bearings

$$R_{ss} = \frac{6l\mu}{\left(a+b-4\left(l+r_p\right)\right)h^3}$$

- ❒ The fluid resistance of the rounded corner regions

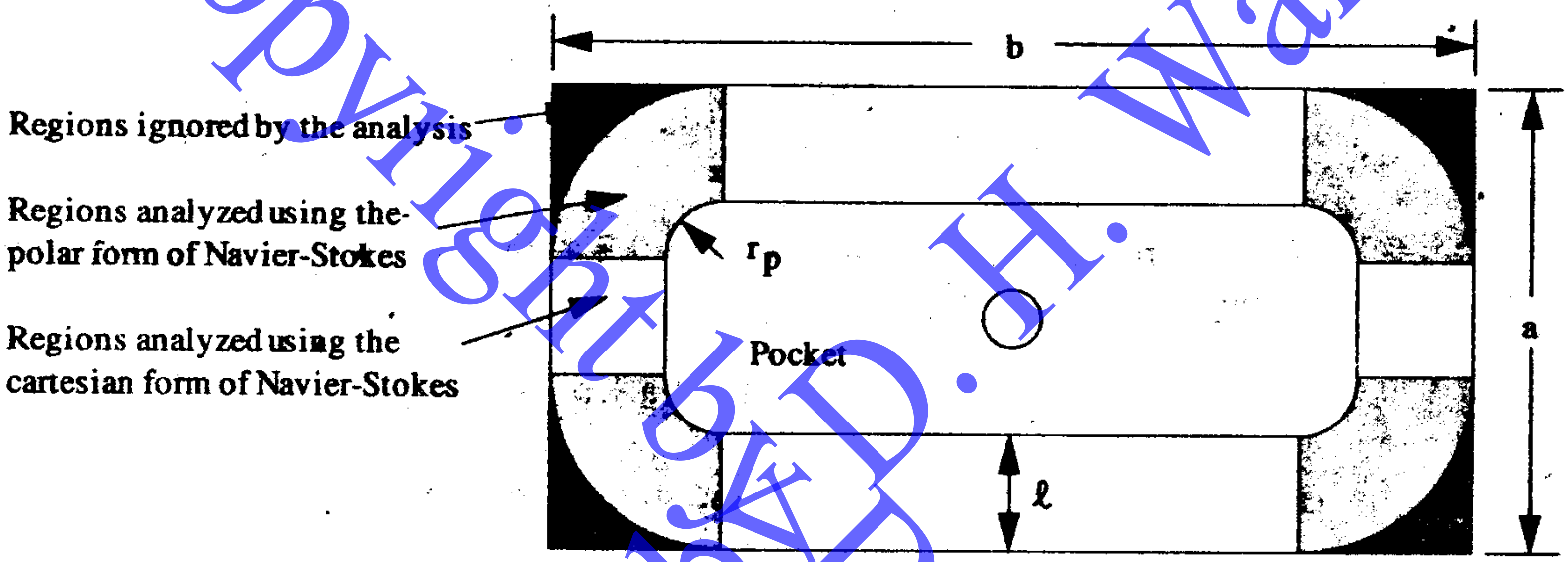
$$R_a = \frac{6\mu\log_e\left(\frac{r_p+\ell}{r_p}\right)}{\pi h^3}$$

Bearing Effective Area

- **The total resistance of the rectangular flat pad pocketed bearing**

$$R = \frac{1}{\frac{1}{R_a} + \frac{1}{R_{ss}}} = \frac{6\mu}{h^3\left[\frac{\pi}{\log_e\left(\frac{r_p + \ell}{r_p}\right)} + \frac{a + b - 4\,(\ell + r_p)}{\ell}\right]}$$

Bearing Effective Area



- The pocketed region contributes a force equal to:

$$F_{pocket} = Pp\left[(a - 2\ell)(b - 2\ell) + r_p^2(\pi - 4)\right]$$

- For the land regions not at the corners, the pressure decays linearly and they contribute a force of:

$$F_{land} = Pp\,\ell\left[a + b - 4(\ell + r_p)\right]$$

Bearing Effective Area

- For the corner regions, the pressure decays logarithmically and the four corners together act as a single beveled ring:

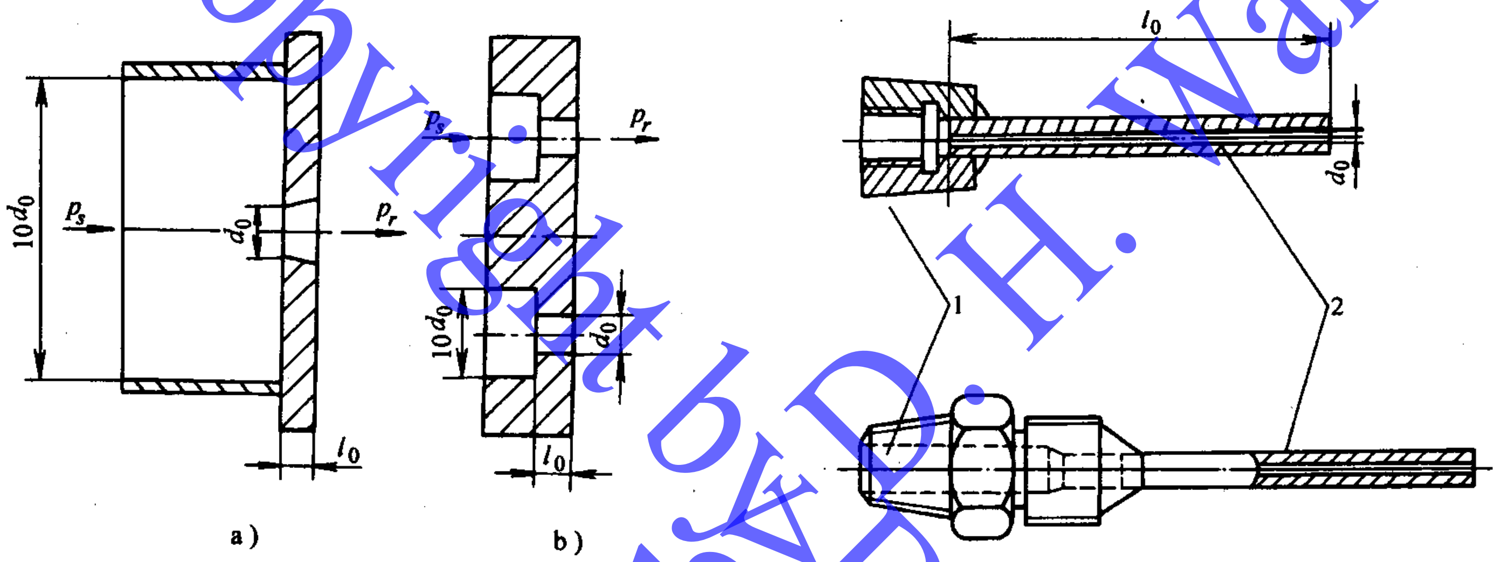
$$F_{rc} = \frac{-2\pi P_p}{\log_e\left(\frac{r_p+\ell}{r_p}\right)} \int_{r_p}^{r_p+\ell} r \log_e\left(\frac{r}{r_p+\ell}\right) dr = \pi P_p \left[\frac{\ell\,(2r_p+\ell)}{2\log_e\left(\frac{r_p+\ell}{r_p}\right)} - r_p^2 \right]$$

- The effective area for the rectangular flat pad pocketed bearing is:

$$A = (a-2\ell)(b-2\ell) + r_p^2(\pi-4) + \ell\left[a+b-4(\ell+r_p)\right] + \pi\left[\frac{\ell\,(2r_p+\ell)}{2\log_e\left(\frac{r_p+\ell}{r_p}\right)} - r_p^2 \right]$$

- Let $r_p = 0.4142\,l$, then $A = ab - l(b+a)$

Flow Restrictor Design



☐ Orifices

☐ Flat-edge pins

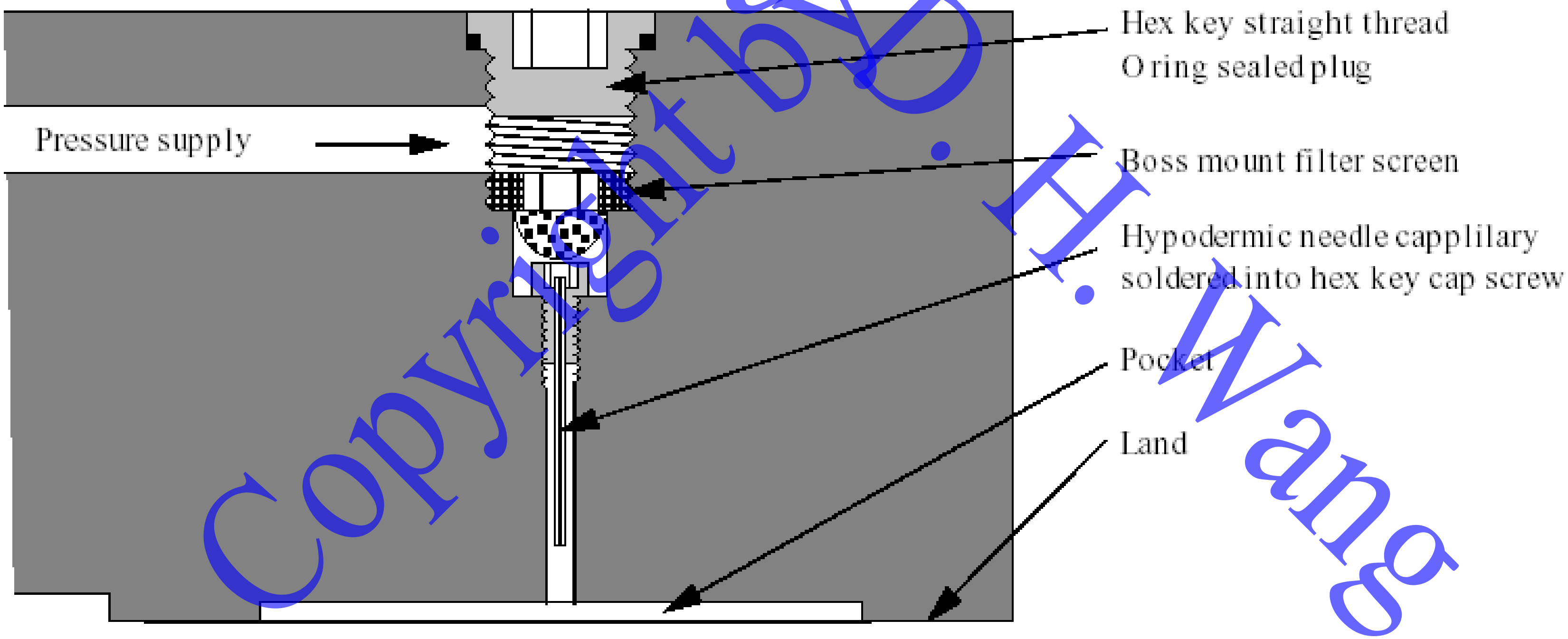
☐ Capillary tubes

☐ Constant-flow devices

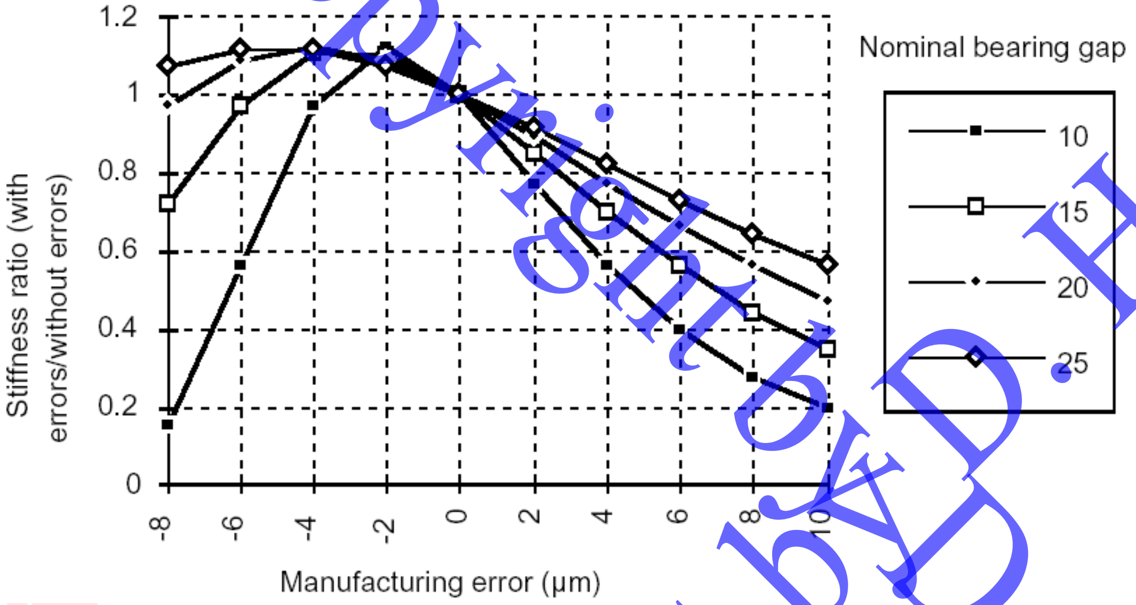
☐ Proportional flow restrictors

Capillary Tubes

- Types of compensation
- Opposed pad, capillary restricted bearings are one of the most common hydrostatic bearing designs.
- The flow resistance of a capillary is:
- Typical design:



Capillary Tubes

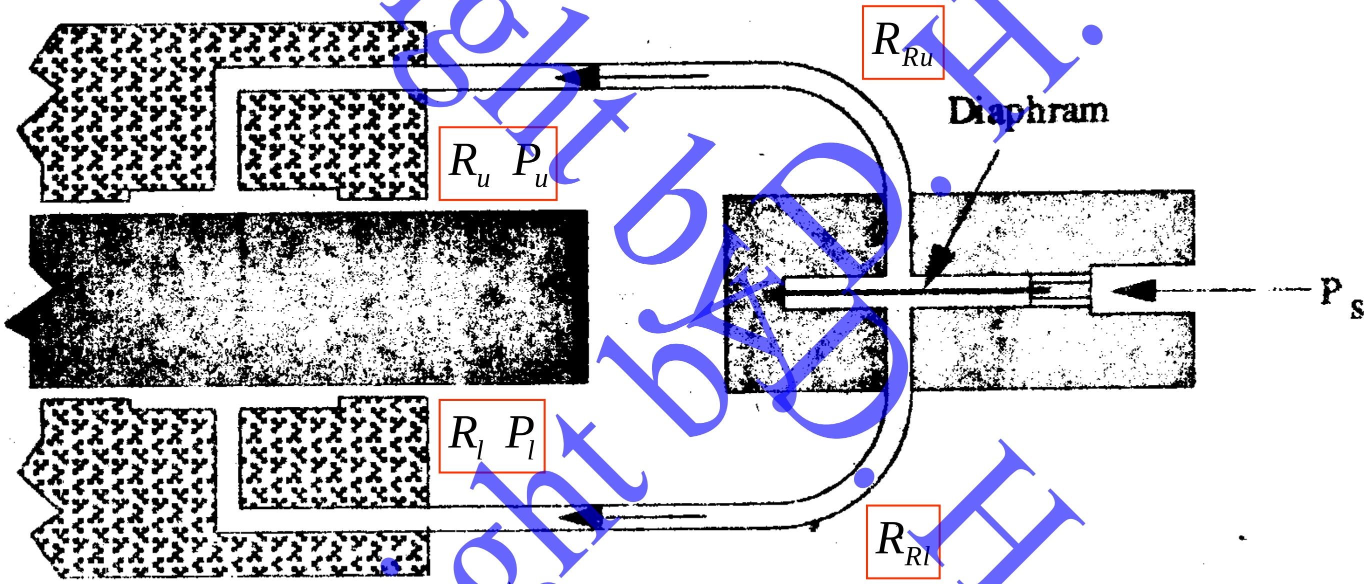


❐ If the gap should change significantly, however, stiffness can be rapidly lost.

❐ Effect of mfg. errors on capillary compensated bearing:

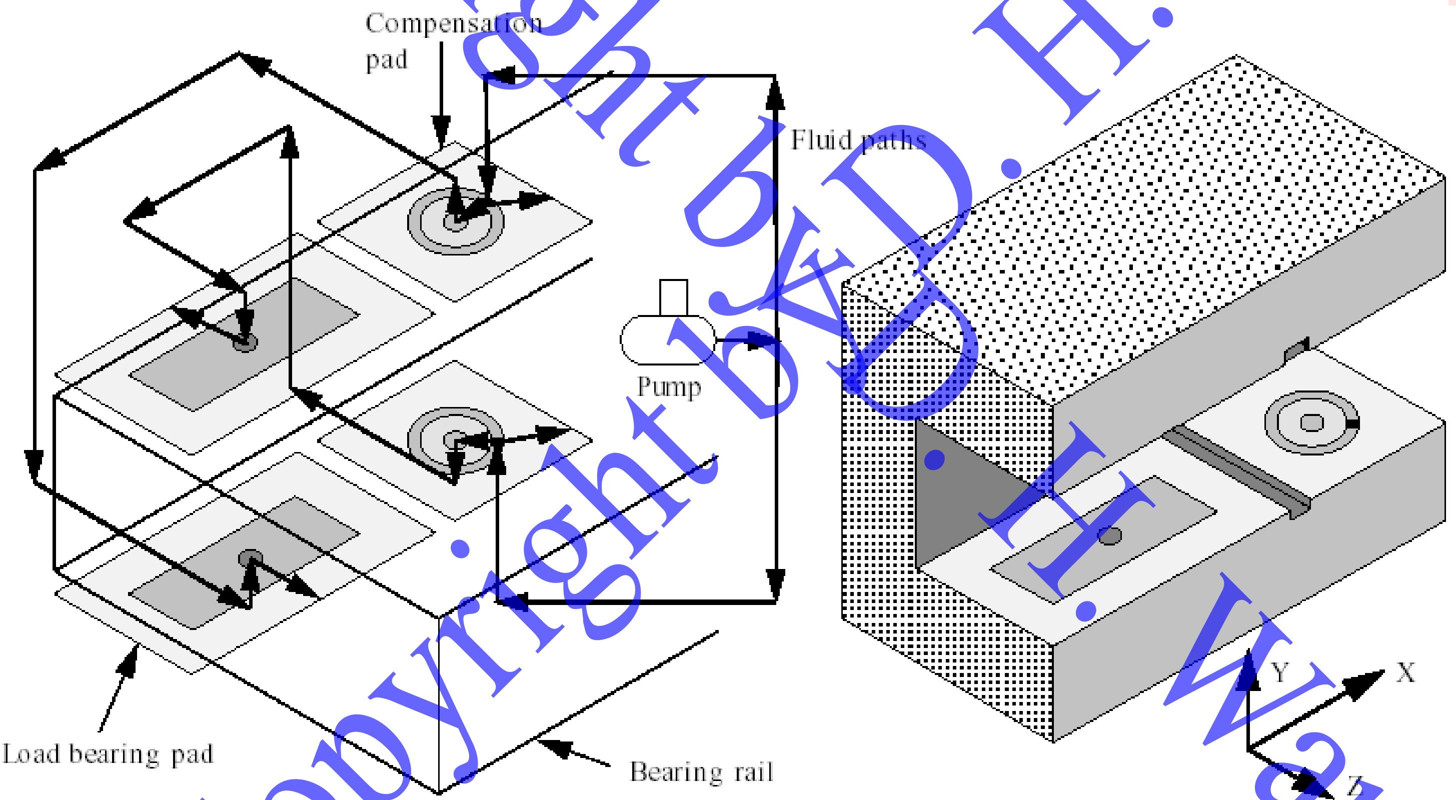
Proportional Flow Restrictor

- Operating principle of Diaphragm-type flow restrictor

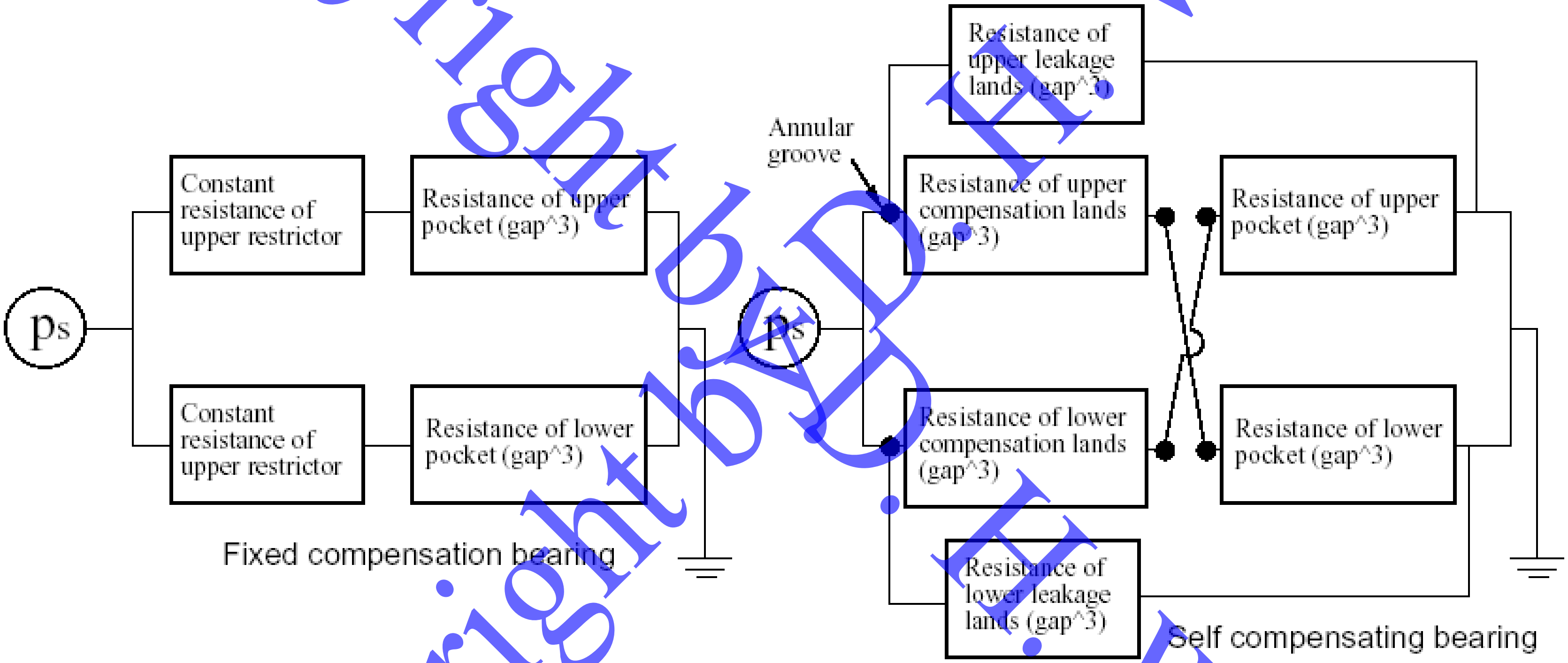


Self-Compensating Bearings

- The bearing gap itself can be used as a means of regulating the flow of fluid to the opposed bearing.



Self-Compensating Bearings



ps

Constant resistance of upper restrictor

Resistance of upper pocket (gap^3)

Constant resistance of upper restrictor

Resistance of lower pocket (gap^3)

Fixed compensation bearing

ps

Annular groove

Resistance of upper leakage lands (gap^3)

Resistance of upper compensation lands (gap^3)

Resistance of upper pocket (gap^3)

Resistance of lower compensation lands (gap^3)

Resistance of lower pocket (gap^3)

Resistance of lower leakage lands (gap^3)

Self compensating bearing

Self-Compensating Bearings

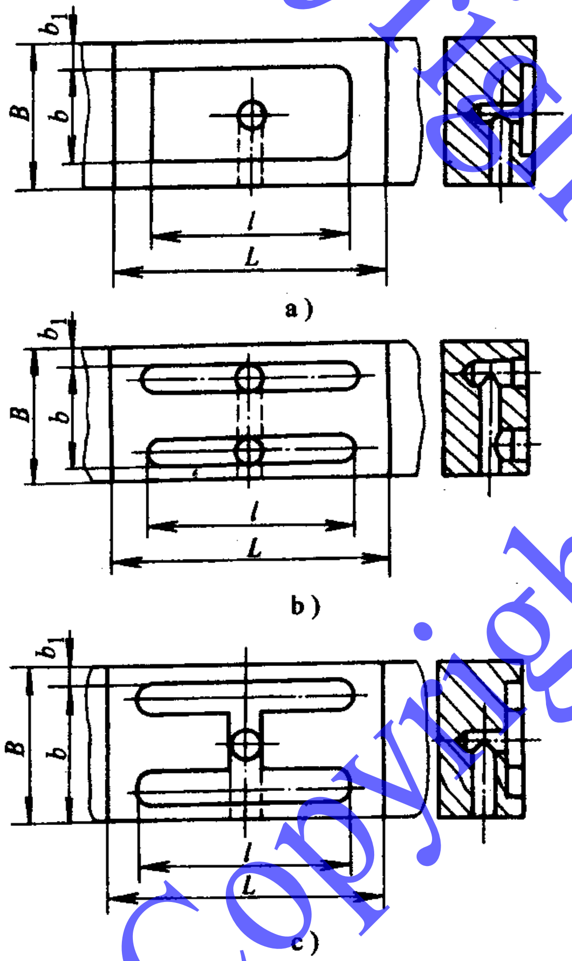
- ❐ **Self compensating bearings' load capacity and stiffness can be theoretically determined:**

$$\Delta P = P_s \left(\frac{\frac{1}{(h-\delta)^3}}{\frac{\gamma}{(h+\delta)^3} + \frac{1}{(h-\delta)^3}} - \frac{\frac{1}{(h+\delta)^3}}{\frac{\gamma}{(h-\delta)^3} + \frac{1}{(h+\delta)^3}} \right)$$

$$K = 3A\,P_s \left\{ \frac{\frac{\gamma}{(h-\delta)^4} - \frac{1}{(h+\delta)^4}}{(h+\delta)^3 \left(\frac{\gamma}{(h-\delta)^3} + \frac{1}{(h+\delta)^3} \right)^2} + \frac{1}{(h+\delta)^4 \left(\frac{\gamma}{(h-\delta)^3} + \frac{1}{(h+\delta)^3} \right)} \right.$$
$$\left. - \frac{\frac{1}{(h-\delta)^4} - \frac{\gamma}{(h+\delta)^4}}{(h-\delta)^3 \left(\frac{1}{(h-\delta)^3} + \frac{\gamma}{(h+\delta)^3} \right)^2} + \frac{1}{(h-\delta)^4 \left(\frac{1}{(h-\delta)^3} + \frac{\gamma}{(h+\delta)^3} \right)} \right\}$$

Self-Compensating Bearings

- Configuration of pockets



Characteristics

- ❒ **Speed and acceleration limits**
  - --
- ❒ **Applied loads**
  - **Large surface area allows for high load capacity.**
  - **Virtually insensitive to crashes.**
- ❒ **Accuracy**
  - **Axial: limited only by the drive system.**
  - **Lateral: limited by the rails and isolation from the pressure source.**
- ❒ **Preload**
  - **Most designs are inherently preloaded.**
- ❒ ..

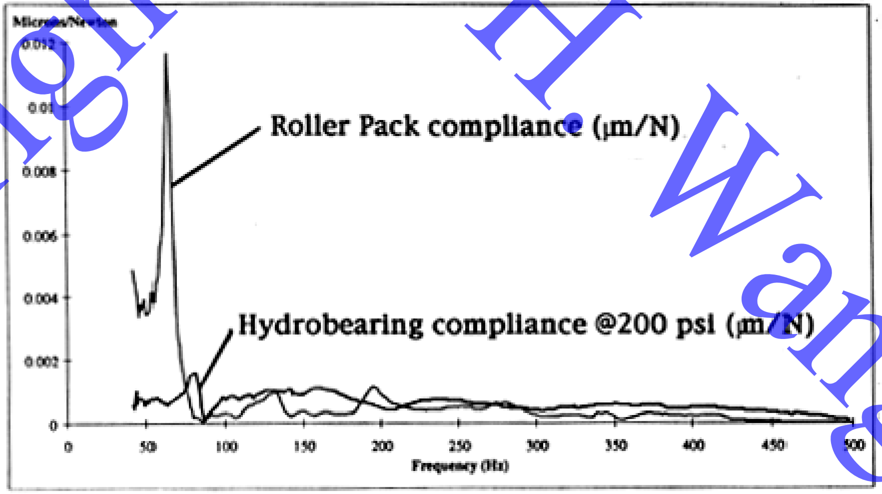
Characteristics

- ❒ **Stiffness**
  - ✐ Easily made many times greater than other components in the machine.
  - ✐ Dynamic stiffness is very high due to squeeze film damping.
- ❒ **Vibration and shock resistance**
  - ✐ Excellent for liquid bearings.
  - ✐ Modest-to-poor for gas bearings.
- ❒ **Damping capability**
  - ✐ …

Characteristics

❒ **Damping capability**

- Excellent normal to direction of motion, due to squeeze film damping.
- Low along direction of motion.
- Bearing area, gap, and stiffness must be considered to maximize squeeze film damping.
- Squeeze film damping greatly affects the dynamic stiffness.



Characteristics

- ❒ **Friction**
  - Zero static friction.
  - Dynamic friction depends on gap and fluid viscosity.
- ❒ **Thermal performance**
  - Finite dynamic friction coefficient generates heat.
  - Fluid flowing at pressure released to atmospheric pressure shears and generates heat equal to pump power.
  - A cooler is often needed to control fluid temperature.
  - Expanding gas creates cooling (Joule Thompson cooling).

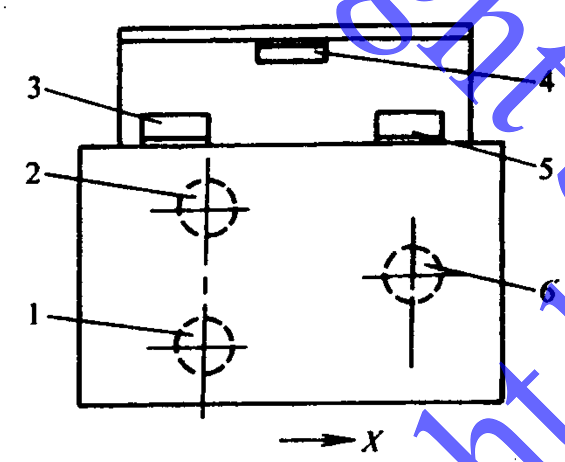
Characteristics

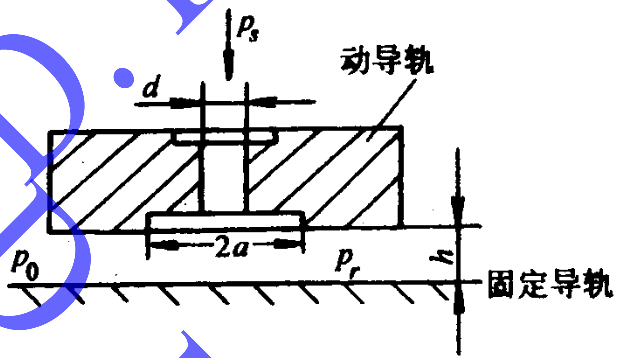
❒ **Environmental sensitivity**

❒ **Support equipment**

Operation Principle

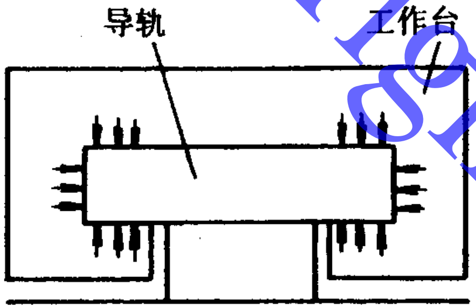
❒ **Length of moving slideway**

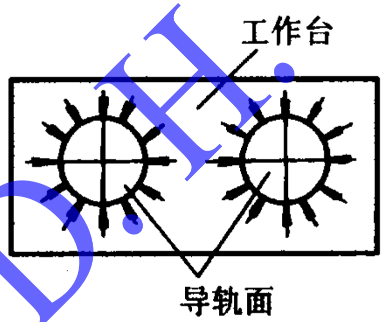


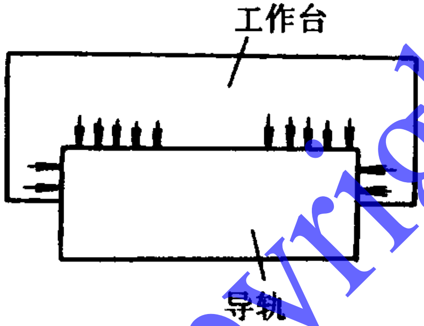


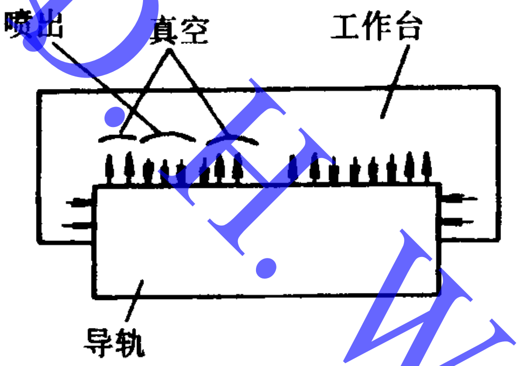
Operation Principle

❒ Length of moving slideway

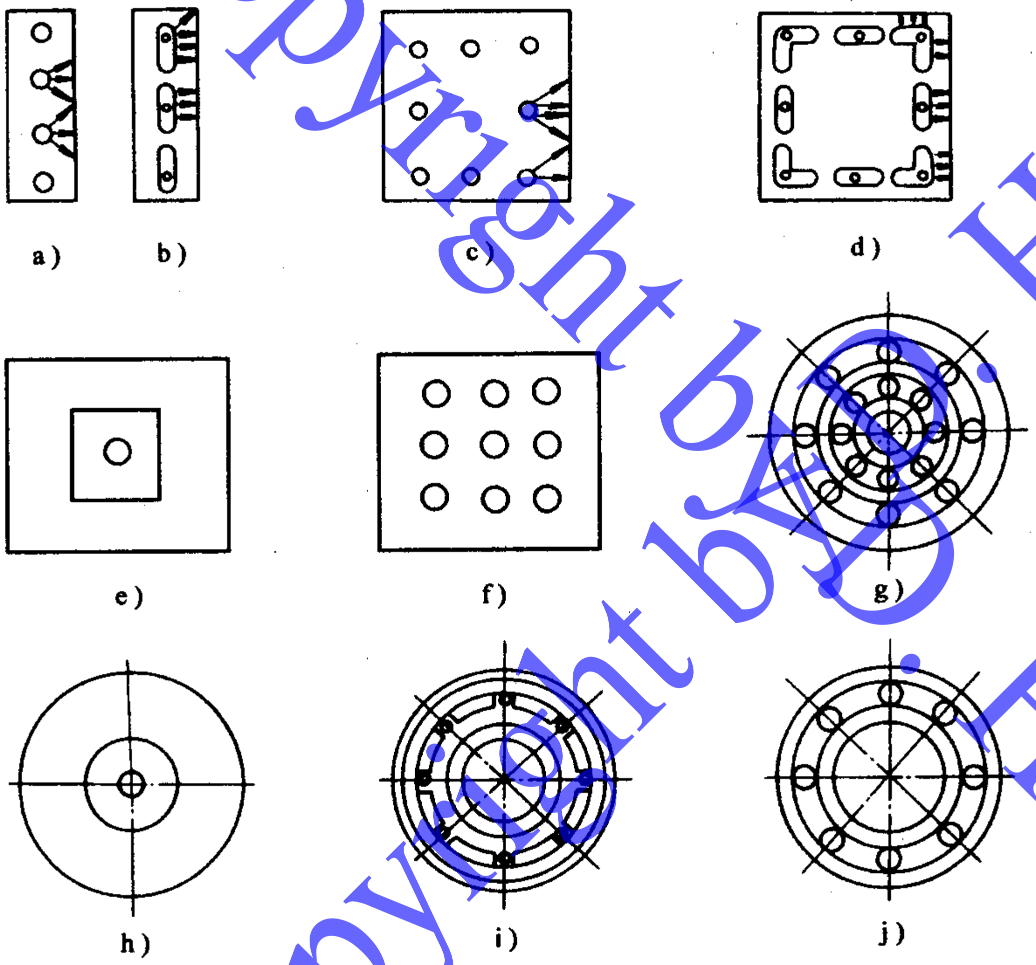








Flow Restrictor Holes

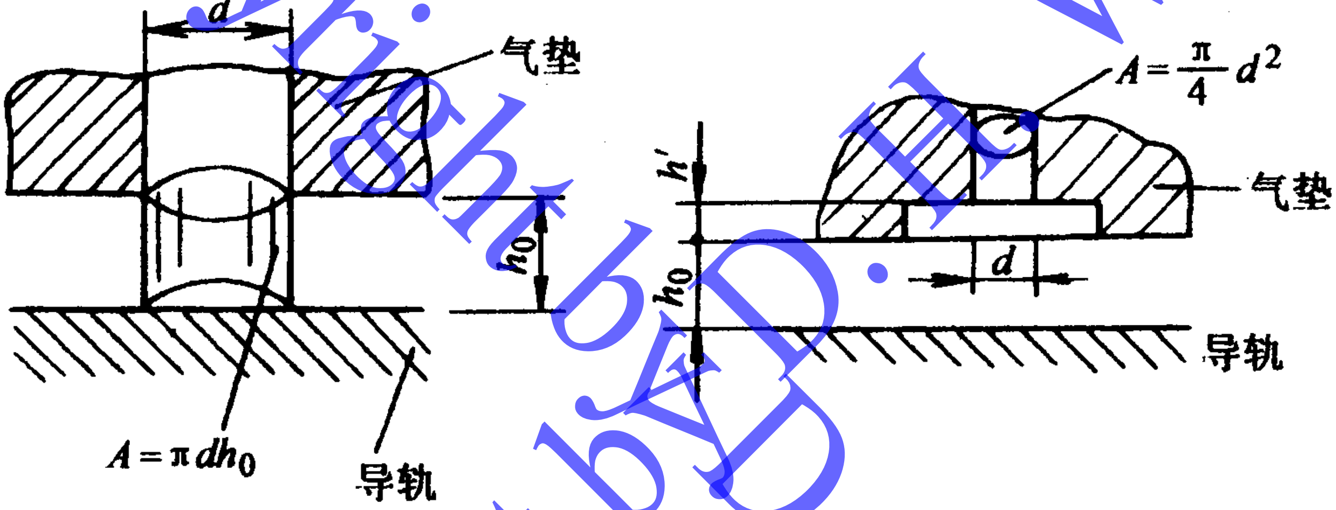


a) b) c) d)

e) f) g)

h) i) j)

Flow Restrictor Holes



a)

b)

Acknowledgement

Thank you very much for your attention !