Lecture 4:
Spindles and Design

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Outline

- ❒ Components of Spindle System
- ❒ Usage
- ❒ Type of Spindles
- ❒ Design Requirements
  - Accuracy
  - Stiffness
  - Vibration
  - Thermal stability
  - Min Deflection
  - Life-span
- ❒ Configurations

Components of Spindle System

Usage

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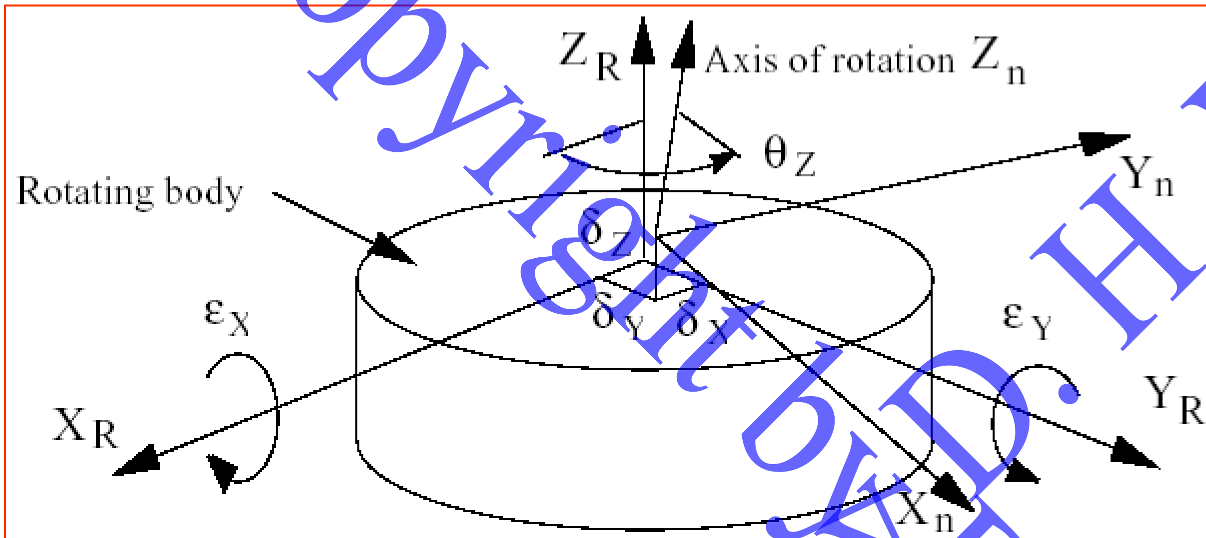
Types of Spindles

- ❒ Sliding contact spindles
- ❒ Rolling element spindles
  - Standard rolling bearing spindle system
  - Non-standard rolling bearing spindle system
    - Single row ball bearings
    - Compact ball bearings
- ❒ Aerostatic spindles
- ❒ Hydrostatic spindles
- ❒ Hydrodynamic spindles
- ❒ Elastic bearings

Design Requirements

- ❒ Accuracy
- ❒ Stiffness
- ❒ Vibration
- ❒ Thermal stability
- ❒ Min Deflection
- ❒ Life-span

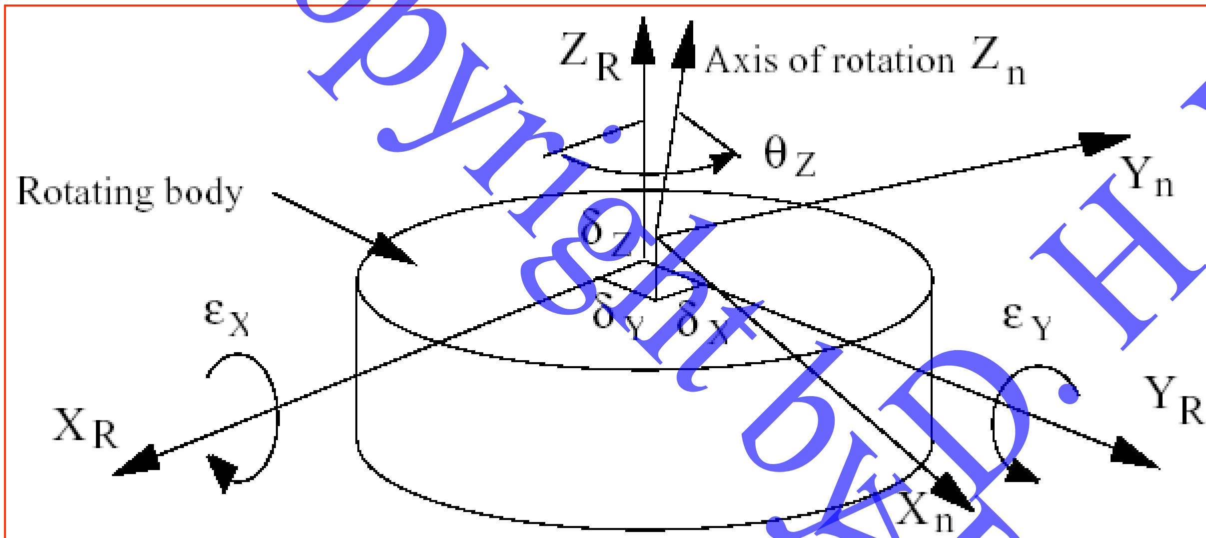
Accuracy



- Motion and errors about an axis of rotation

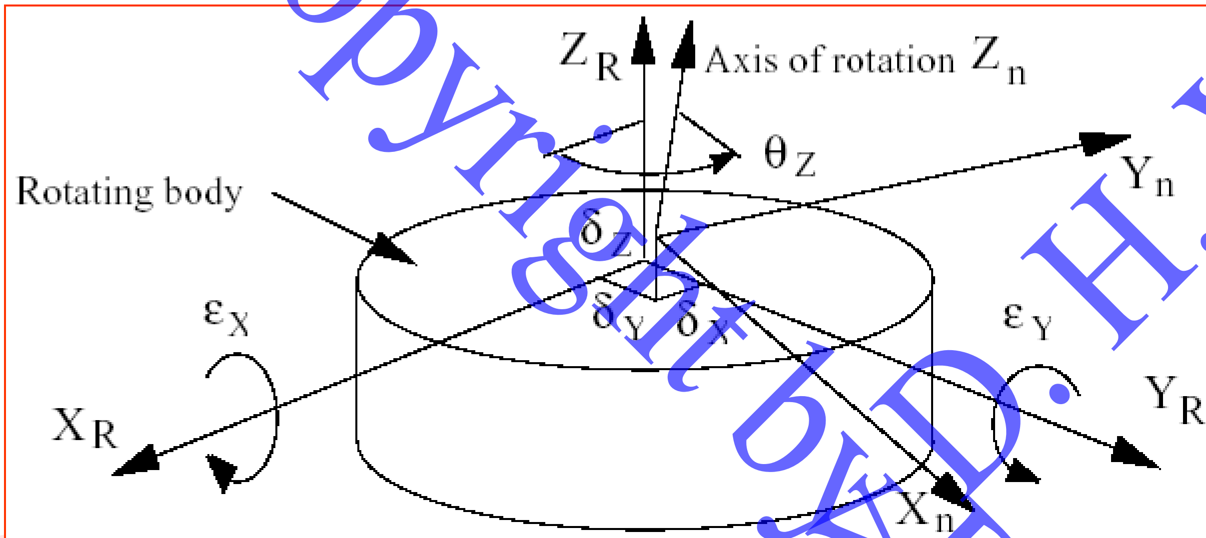
$$
{}^{R}\mathbf{T}_{nerr} = \begin{bmatrix} C\varepsilon_Y C\theta_Z & -C\varepsilon_Y S\theta_Z & S\varepsilon_Y & \delta_X \\ S\varepsilon_X S\varepsilon_Y C\theta_Z + C\varepsilon_X S\theta_Z & C\varepsilon_X C\theta_Z - S\varepsilon_X S\varepsilon_Y S\theta_Z & -S\varepsilon_X C\varepsilon_Y & \delta_Y \\ -C\varepsilon_X S\varepsilon_Y C\theta_Z + S\varepsilon_X S\theta_Z & S\varepsilon_X C\theta_Z + C\varepsilon_X S\varepsilon_Y S\theta_Z & C\varepsilon_X C\varepsilon_Y & \delta_Z \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

Accuracy



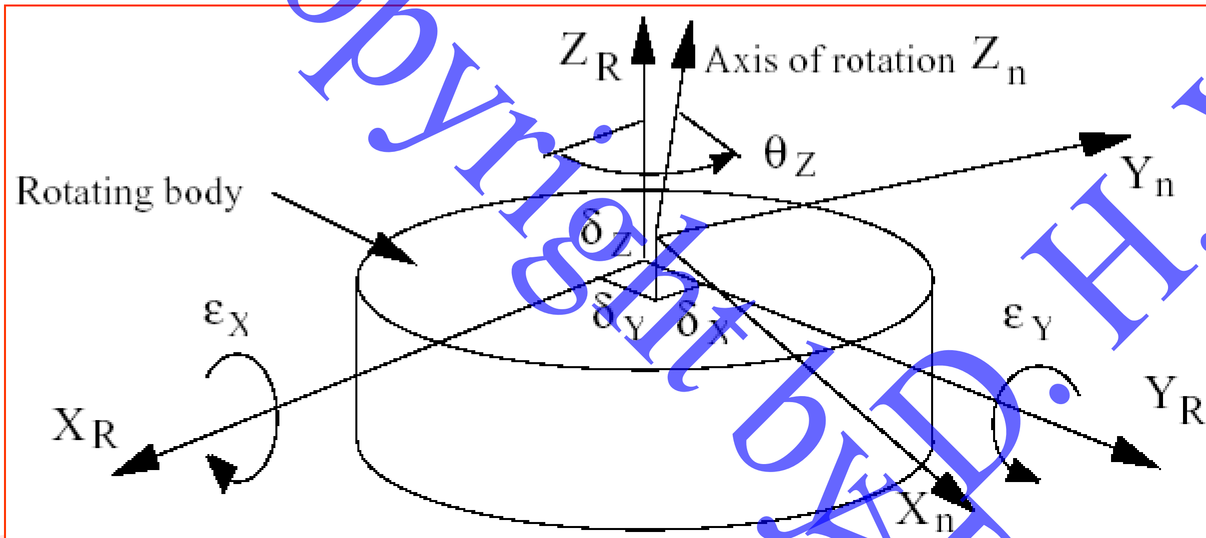
- Axis of rotation: a line about which rotation occurs
- Spindle: a device which provides an axis of rotation
- Perfect spindle: a spindle having no motion of its axis of rotation relative to the reference coordinate axes
- Perfect workpiece: a rigid body having a perfect surface of revolution about a center line

Accuracy



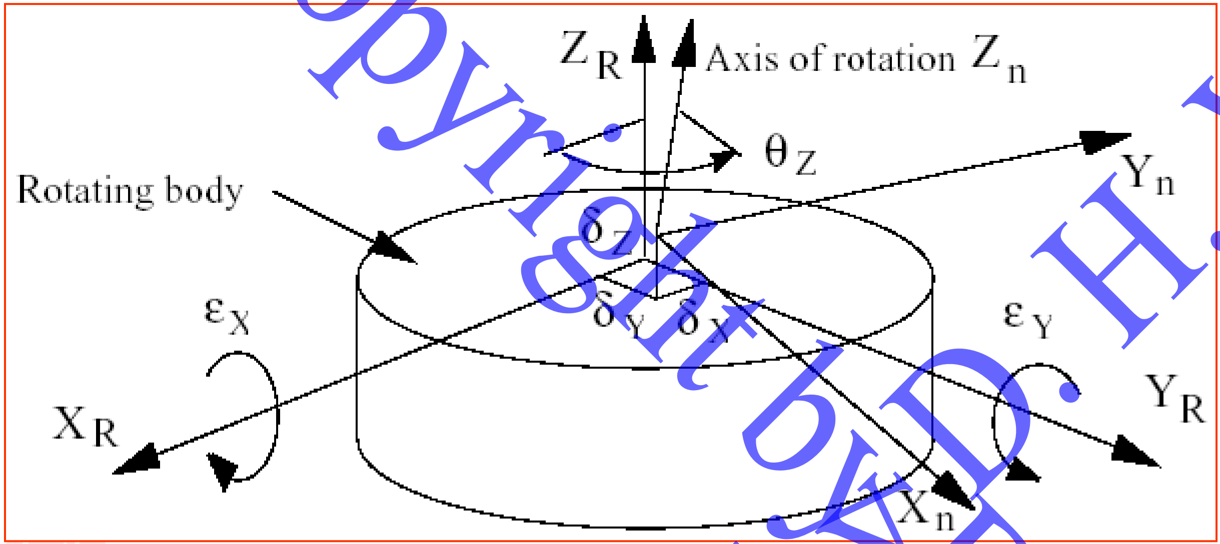
- Error motion: changes in position, relative to the reference coordinate axes, of the surface of a perfect workpiece with its center line coincident with the axis of rotation
- Sensitive and nonsensitive directions: the sensitive direction is perpendicular to the ideal generated workpiece surface through the instantaneous point of

Accuracy



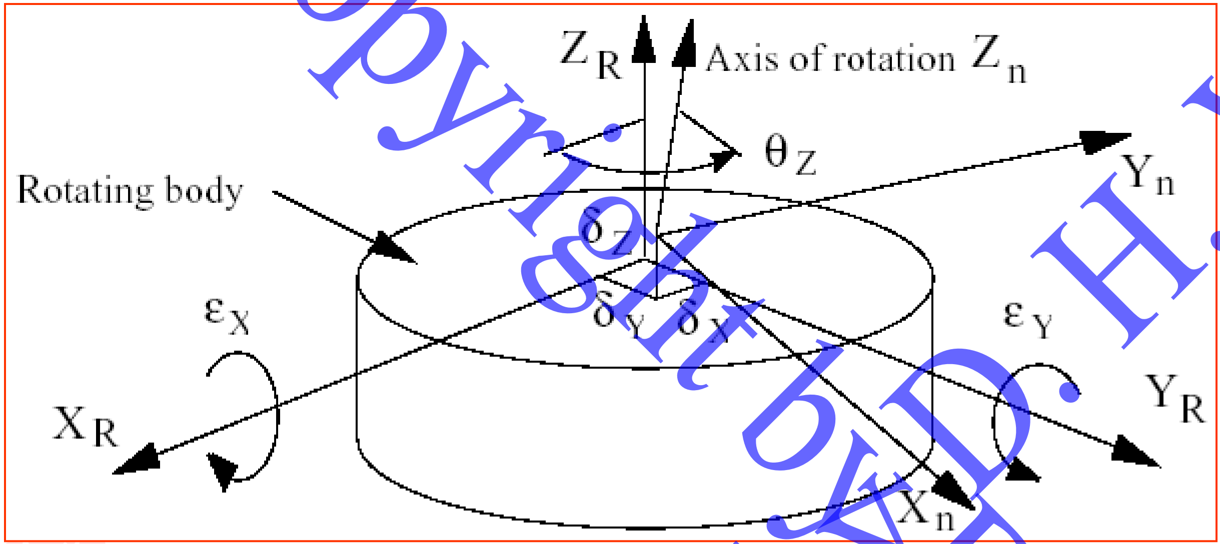
- ❒ Radial motion: error motion in a direction normal to the Z reference axis and at a specified axial location
- ❒ Runout: the total displacement measured by an instrument sensing against a moving surface or moved with respect to a fixed surface
- ❒ Axial motion: error motion colinear with the Z reference axis

Accuracy



- Tilt motion: error motion in an angular direction relative to the Z reference axis
- Face motion: error motion parallel to the Z reference axis at a specified radial location
- Pure radial motion: the concept of radial motion in the absence of tilt motion
- Causes induced error motions of spindles ?

Accuracy



❐ How to select the rotation accuracy types (axial, radial, and/or tilt) of spindle for an instrument ?

- Ex: Spindles of Roundness measuring machines, Spindles of angular meters
- Ex: Spindles of Gyroscopes

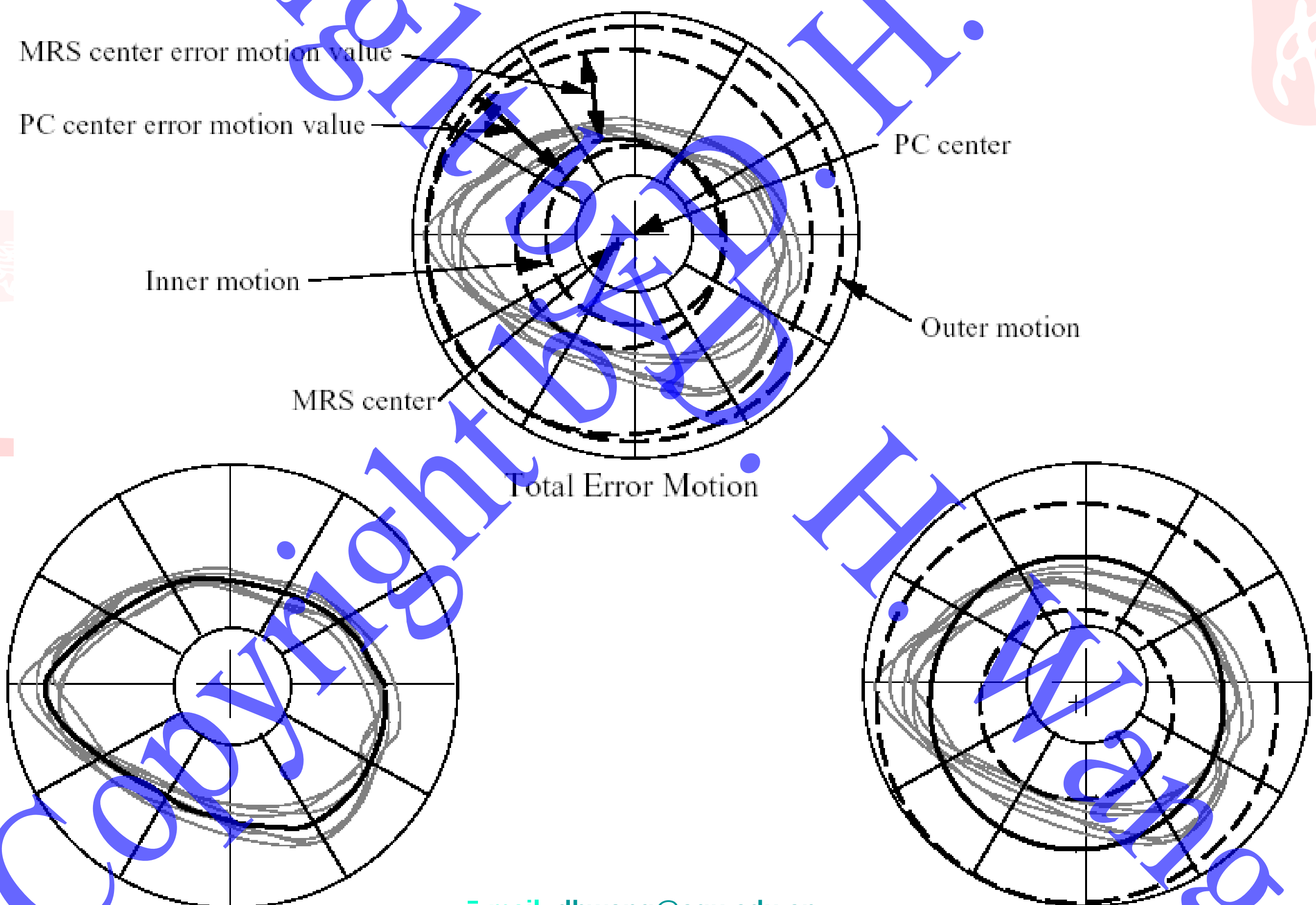
Accuracy: Error Motion Plot

❒ **Error motion polar plot**

- **A polar plot of error motion made in synchronization with the rotation of the spindle**
  - **Total error motion polar plot: the complete error motion polar plot as recorded**
  - **Average error motion polar plot: the mean contour of the total error motion polar plot averaged over the number of revolutions**
  - **Fundamental error motion polar plot: the best-fit reference circle fitted to the average error motion polar plot**

Accuracy: Error Motion Plot

- Examples of error motion and error motion component polar plot

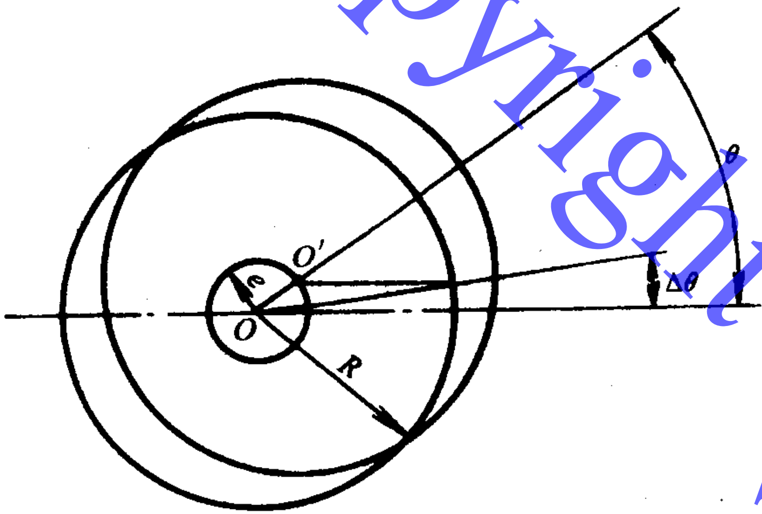


Total Error Motion

Average Error Motion

Fundamental Error Motion

Accuracy



- Clearance and Eccentricity

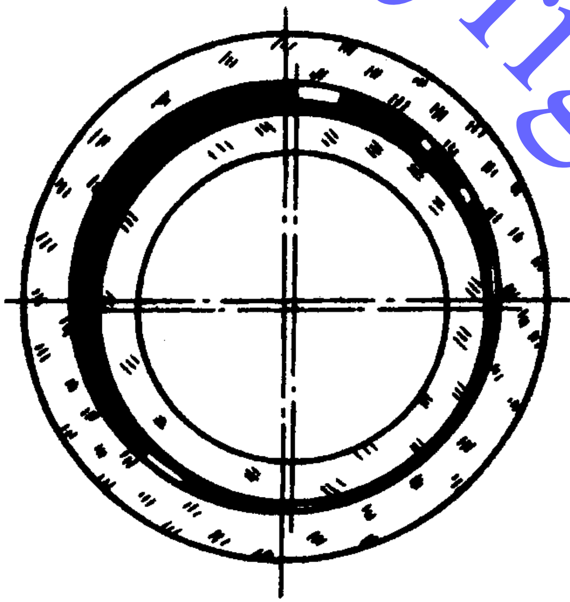
$$e = \frac{\Delta}{2}$$

- Error induced by Eccentricity of shaft and clearance

$$\Delta\theta = \frac{e\sin\theta}{R}$$

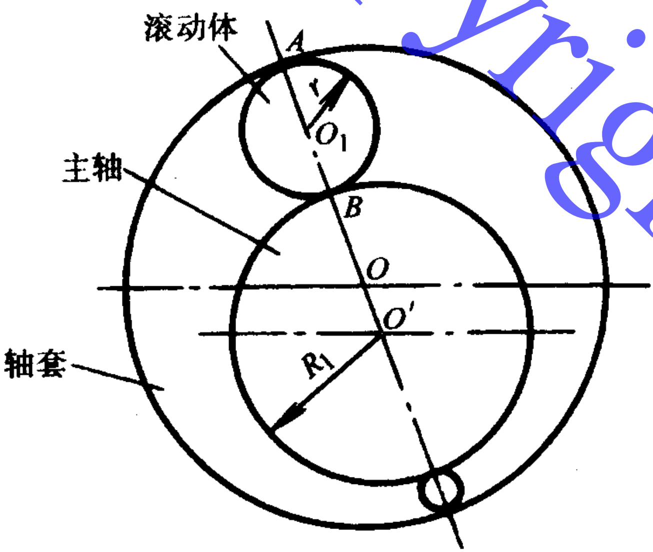
- Wobble and swash of the shaft
  - **Reading head averaging**

Accuracy:双周晃动误差



- ❒ 双周晃动误差
  - 如果在滑动轴系中，在主轴和轴套之间充满润滑油，则主轴回转误差规律与前述不同，这时轴系每转动二周出现误差运动重复一次的现象
- ❒ 滑动轴承的双周晃动误差是由油膜“滚动”引起的。
- ❒ 双周晃动误差不仅有径向误差运动，也会造成倾角误差运动，原因是由于轴系上、下两段轴承油膜厚度方向不可能完全一致。
- ❒ 出现双周晃动误差时滑动轴系在空间运动轨迹大致为单叶旋转双曲面。

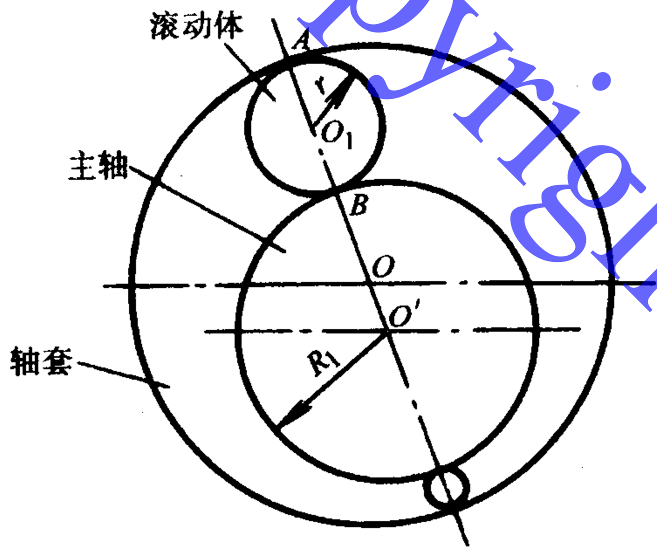
Accuracy: 滚动轴承主轴漂移



$$\theta = \frac{360^\circ s'}{s_{o'o_1}} = \frac{360^\circ \pi R_1}{2\pi(R_1 + r)}$$

- 滚动轴承轴系的运动特点
  - 滚动轴系由主轴、轴套和介于二者之间的滚动体组成，而且是过盈装配做纯滚动运动。
- 漂移
  - 由于轴承内孔、主轴及滚动体形状误差，特别是滚动体有尺寸差，形成了滚动轴系的误差运动。
  - 若轴套固定，主轴旋转，则会产生主轴轴心线位移，称为主轴“漂移”。
- 减小滚动轴系的主轴“漂移”的办法：
  - ……

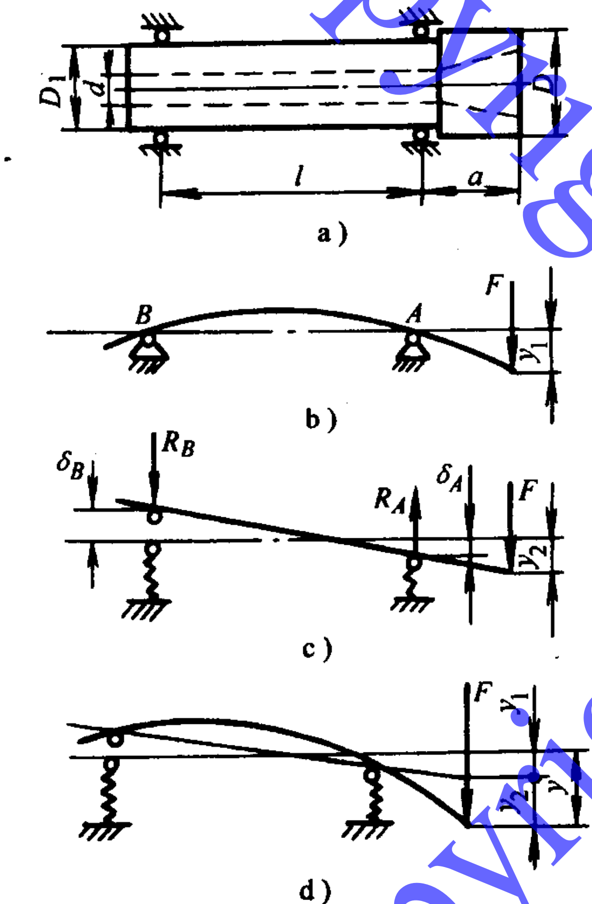
Accuracy: 滚动轴承主轴漂移



$$\theta = \frac{360^\circ s'}{s_{o'o_1}} = \frac{360^\circ \pi R_1}{2\pi(R_1 + r)}$$

- 减小滚动轴系的主轴“漂移”的办法
  - 严格控制主轴、滚动体、轴套的尺寸误差和形状误差，尤其应尽量减小滚动体的尺寸一致性误差。
  - 采用误差平均原理，用平均读数法尽量减小主轴“漂移”带来的读数误差。

Stiffness



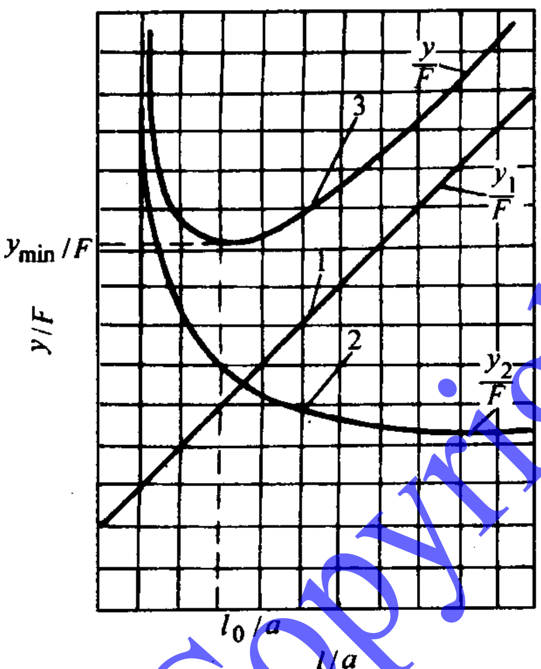
- Sources of deflection in a spindle include contributions from the spindle shaft, bearings, and housing

$$y = y_1 + y_2$$

$$= \frac{Fa^3}{3EJ_1}\left(\frac{J_1}{J_2}+\frac{l}{a}\right)+\frac{F}{K_A}\left[\left(1+\frac{K_A}{K_B}\right)\frac{a^2}{l^2}+\frac{2a}{l}+1\right]$$

❒ Relationship between y/F and l/a

$$y = y_1 + y_2 = \frac{Fa^3}{3EJ_1}\left(\frac{J_1}{J_2} + \frac{l}{a}\right) + \frac{F}{K_A}\left[\left(1 + \frac{K_A}{K_B}\right)\frac{a^2}{l^2} + \frac{2a}{l} + 1\right]$$



Stiffness

❐ Optimal span of bearings

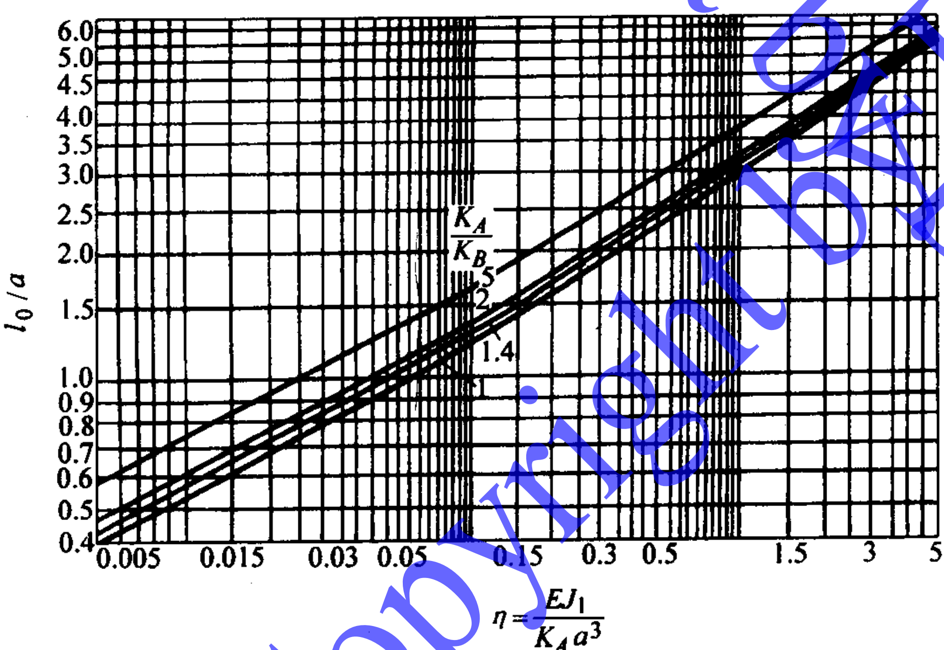
$$y = y_1 + y_2 = \frac{Fa^3}{3EJ_1}\left(\frac{J_1}{J_2}+\frac{l}{a}\right) + \frac{F}{K_A}\left[\left(1+\frac{K_A}{K_B}\right)\frac{a^2}{l^2} + \frac{2a}{l} + 1\right]$$

Let $\frac{\mathrm{d}y}{\mathrm{d}l} = 0$ ➡

$$l_0^3 - \frac{6EJ_1 l}{K_A a} - \frac{6EJ_1}{K_A}\left(1+\frac{K_A}{K_B}\right) = 0$$

$$\eta = \frac{EJ_1}{K_A a^3}$$

$$\eta = \left(\frac{l_0}{a}\right)^3 \frac{1}{6\left(\frac{l_0}{a} + \frac{K_A}{K_B} + 1\right)}$$

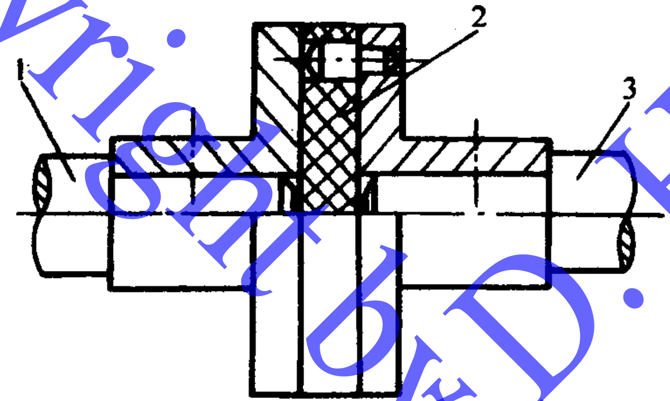


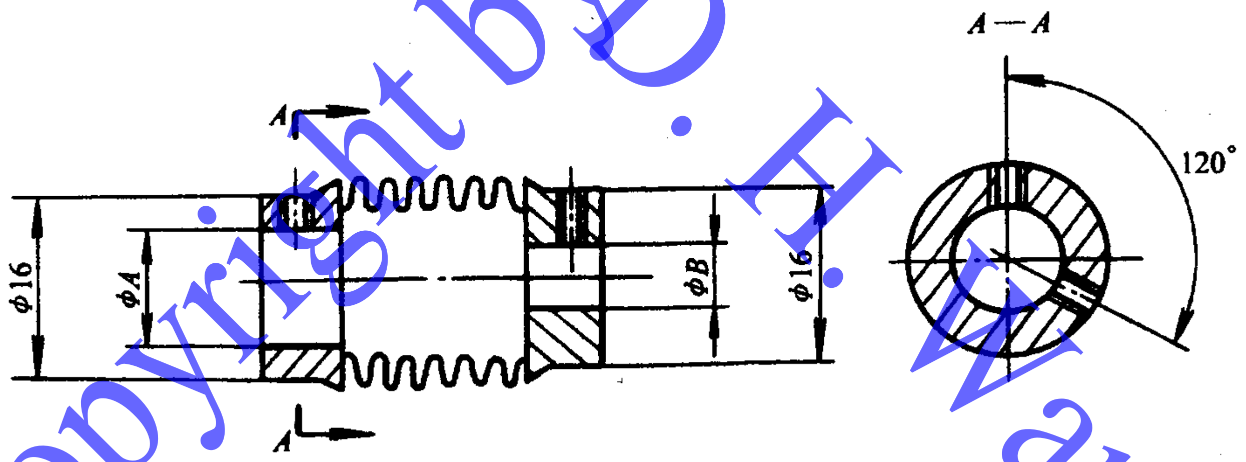
Stiffness

❒ How to improve the stiffness of spindle systems ?

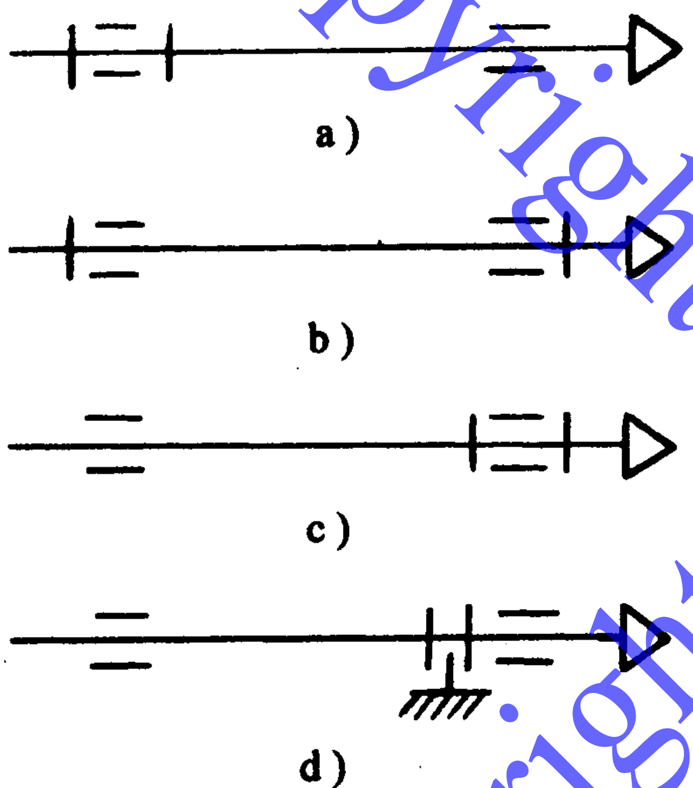
- 加大主轴直径
- 合理选择支承跨距
- 缩短主轴悬伸长度
- 提高轴承刚度

Vibration



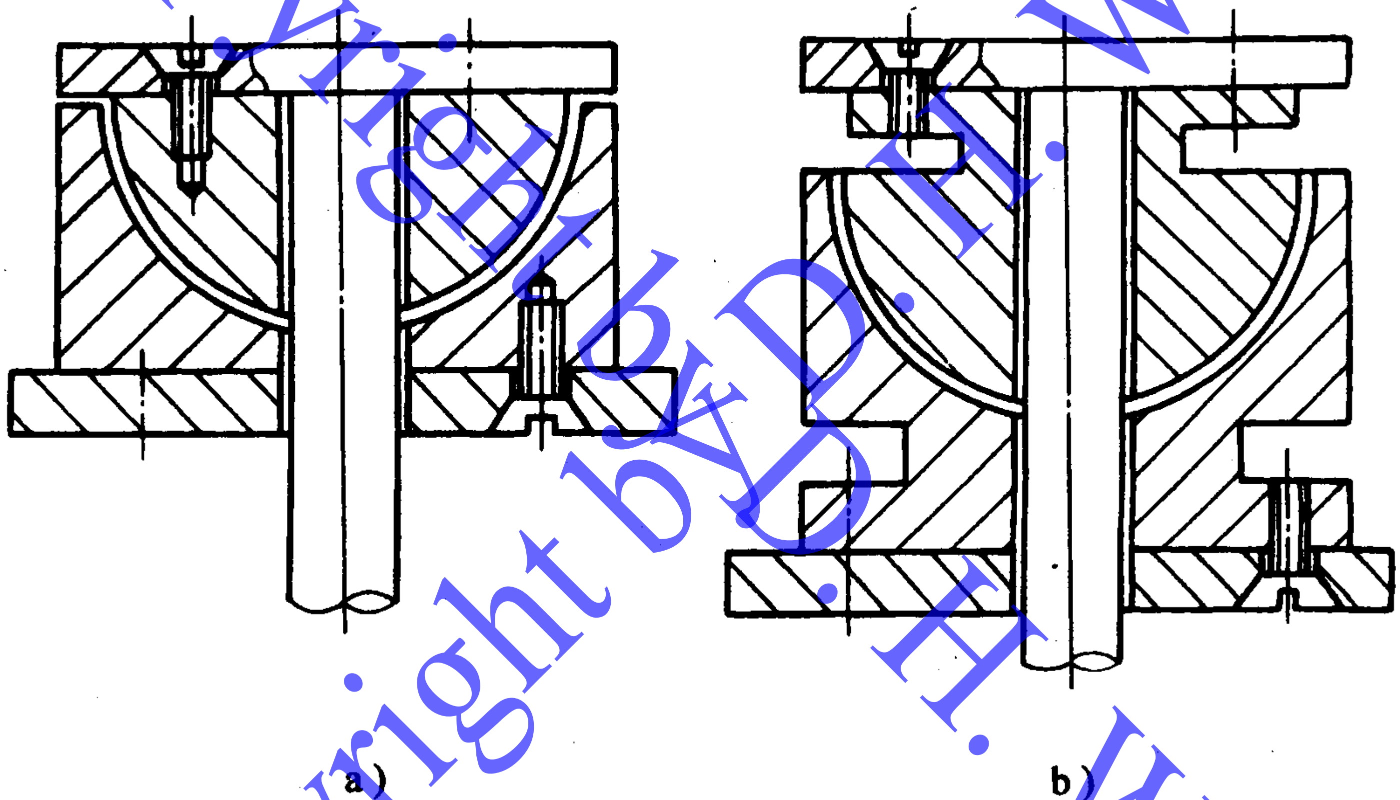


Thermal Expansion



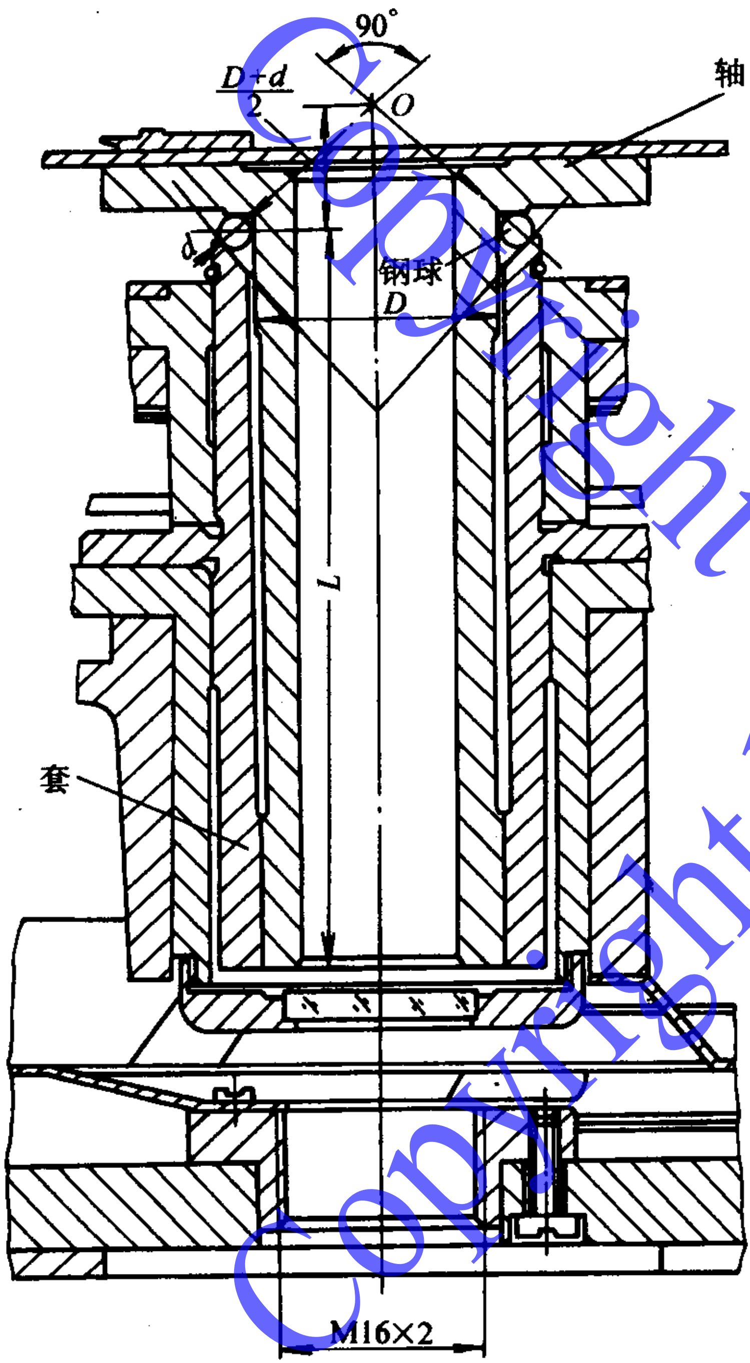
- 正确选择和设计轴系
  - 普通滑动轴系，当轴承间隙小于0.2~0.5 um时，温度不当会产生抱轴
  - 滚动轴系与静压轴系抵御温度变化能力好
- 合理选择推力支承位置
- 减小热源影响
- 采用热补偿措施

Appropriate Mechanical Structure



Life-Span

- Required life
  - Properly designed, manufactured, installed, and maintained:
    - Rolling element bearings can provide tens of millions of cycles of trouble free accurate motion.
    - Assumes proper mounting including manufacturing tolerances.
    - Some wear will occur with every cycle.
    - Elastic preloading compensates for some geometry change.
  - The greatest unknowns are contamination getting by the seals, and impact loading.



- 半运动式圆柱形轴承轴系(经纬仪轴系)
- 等效工作长度

$$L' = L + \frac{(D+d)}{2}$$

- 角运动误差

$$\Delta\phi = \Delta h\rho'' \Big/ \left(L + \frac{D+d}{2}\right) \leq 1''$$

- 标准圆柱式轴系角运动误差

$$\Delta\phi = \Delta h\rho'' \Big/ L$$

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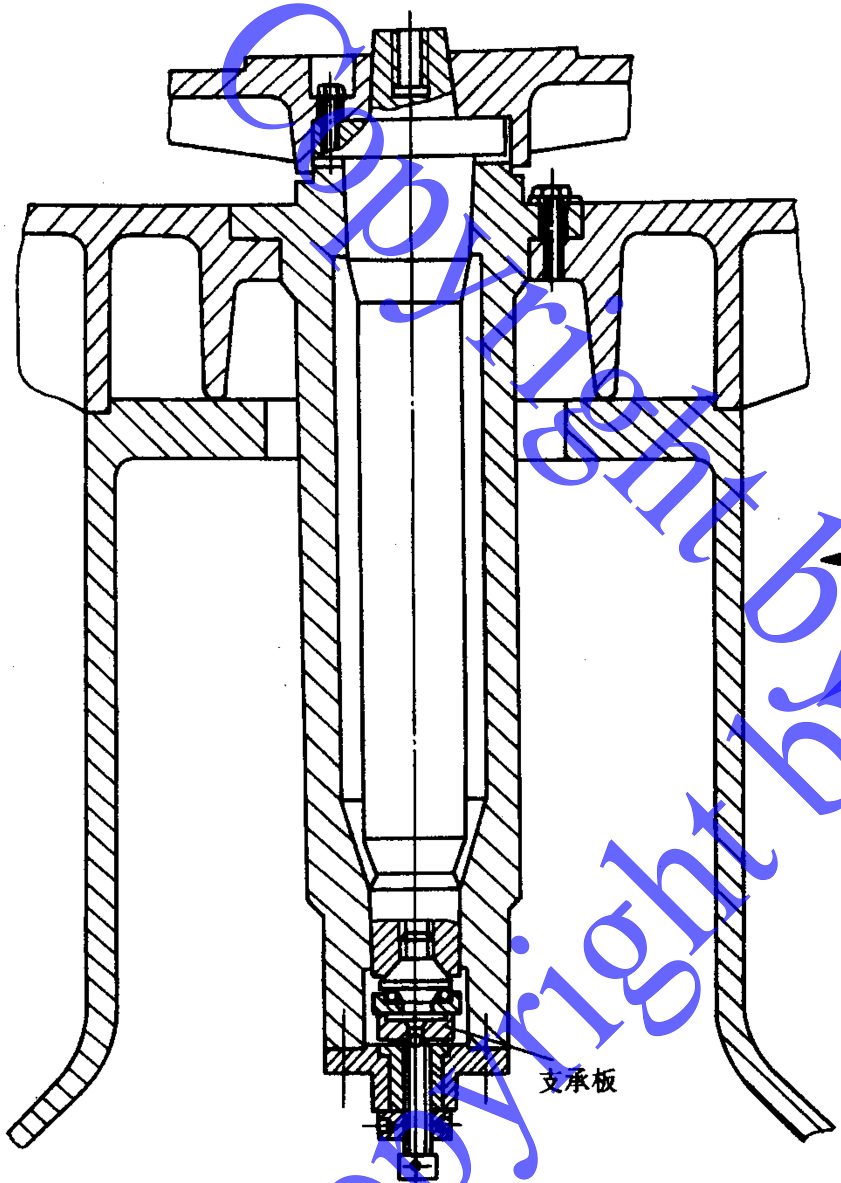
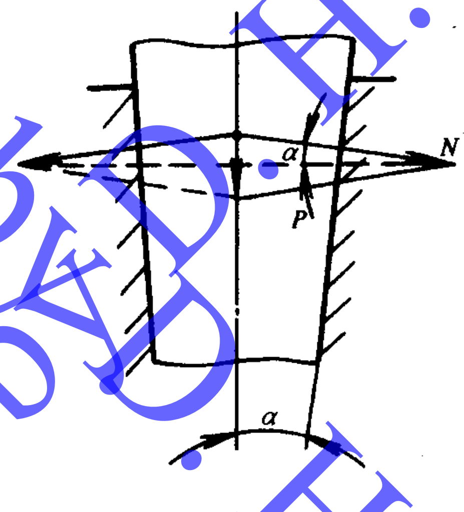
- 圆锥形滑动轴系



a)



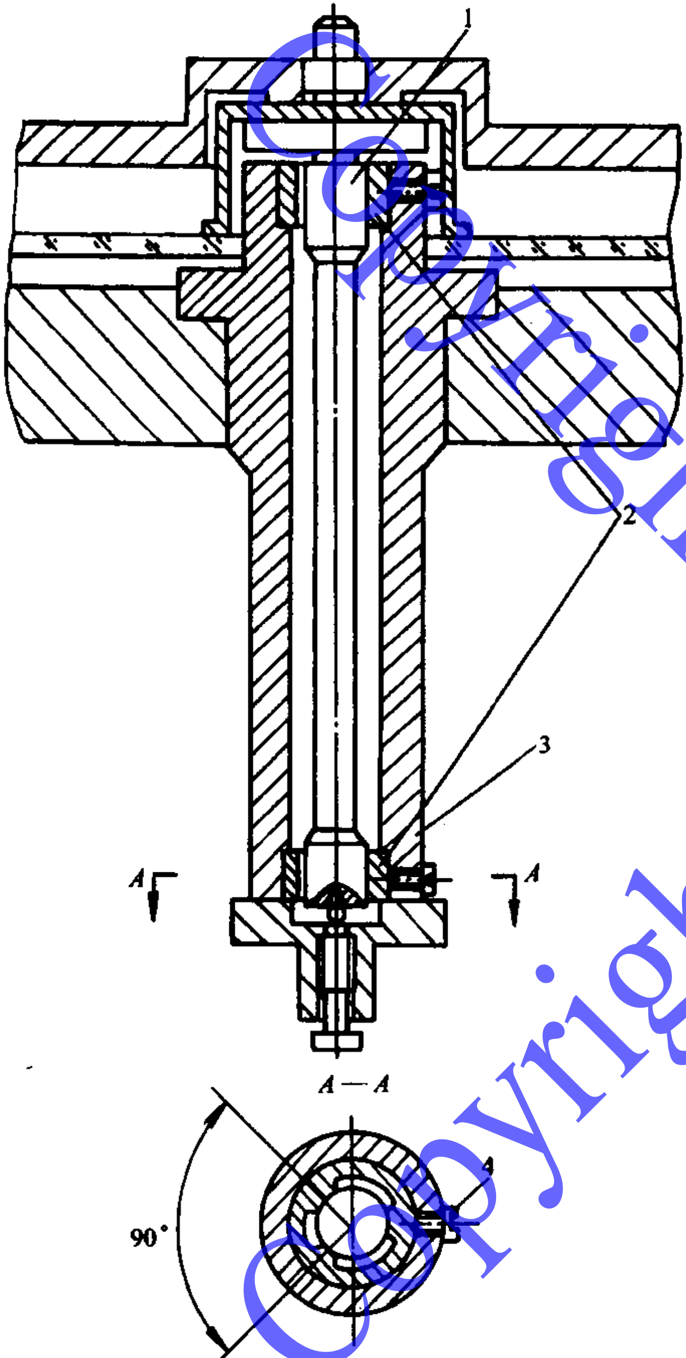
b)

$$N = P \big/ 2\sin\alpha$$

$$4^\circ \le 2\alpha \le 15^\circ$$

$$\Delta\phi = \Delta h \rho'' \cos\alpha \big/ L$$

❒ V形弧滑动轴承轴系



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Rotary Rolling Bearings

❒ **Standard rolling bearing spindle system**

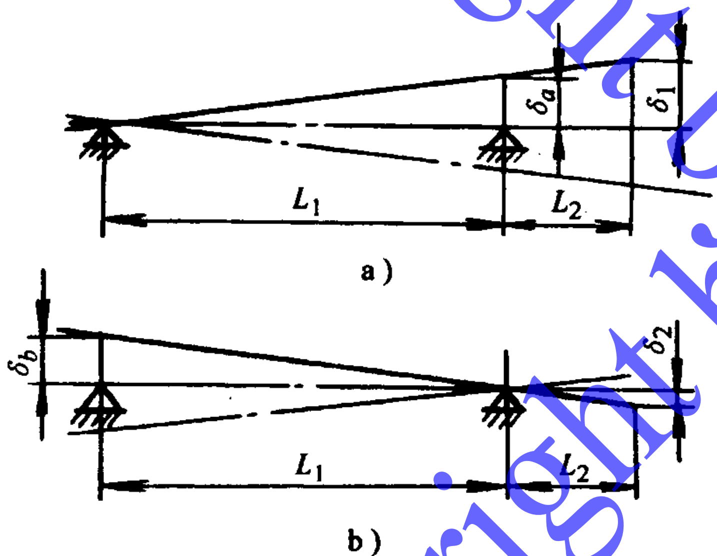
❒ **Non-standard rolling bearing spindle system**

- **Single row ball bearings**
- **Compact ball bearings**

Rolling Element Bearings

❐ **Standard rolling bearing spindle system**

✐ 后轴承的精度通常可以比前轴承的精度可低一级

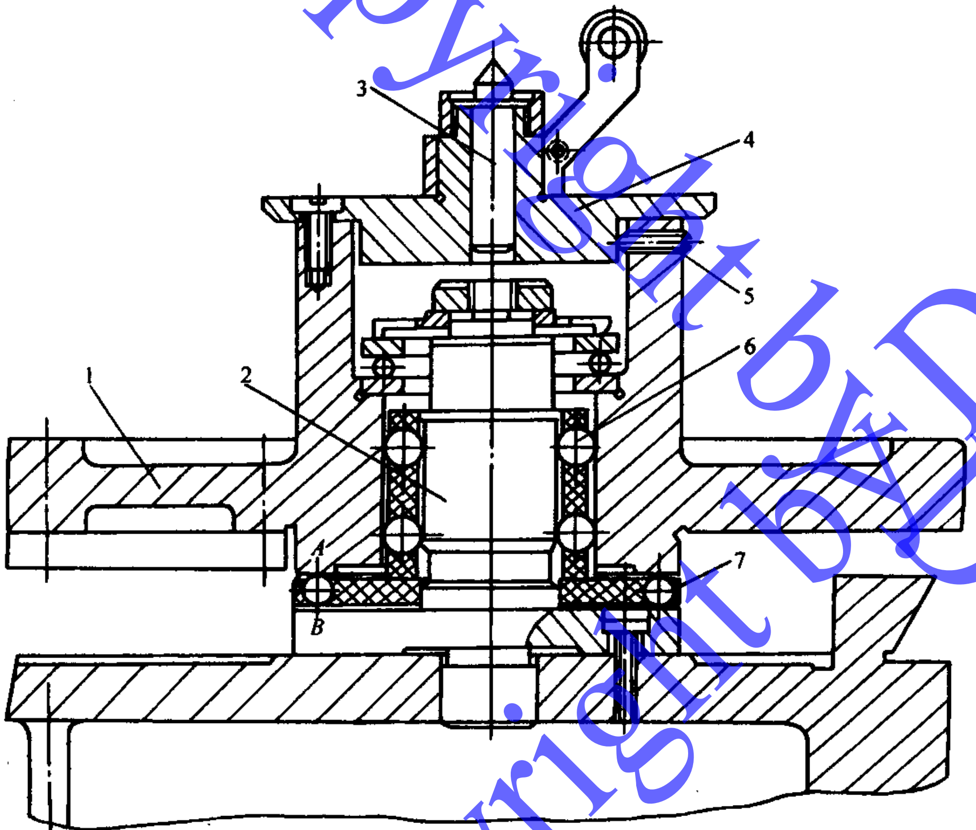


$$\delta_1 = \frac{L_1 + L_2}{L_1 \delta_a}$$

$$\delta_2 = \frac{L_2}{L_1 \delta_b}$$

When $\delta_a = \delta_b$ ， $\delta_1 > \delta_2$

Single Row Ball Bearings--Ex.



- ❐ **Spindle system of involute tooth profile measuring machine**
  - **Radial precision**
  - **Axial precision**

Determine the Major Parameters: Thrust Bearing

❒ **Determine the diameter of balls**

$$d = \sqrt{\frac{W_k}{9.8[P]}}$$

$$W_k = W_0 a_1 a_2 a_3$$

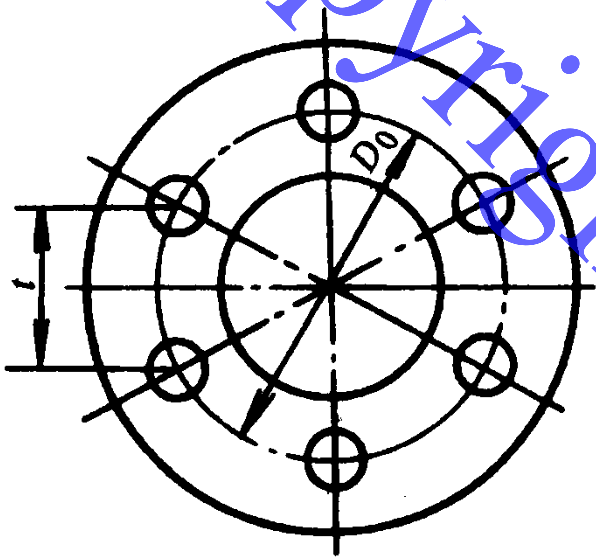
$$W_0 = \frac{W}{zk}$$

$$\delta = 2 \times 0.76 \sqrt[3]{\frac{2W_0^2}{d}\left(\frac{1}{E_1} + \frac{1}{E_2}\right)^2}$$

❒ **Determine the number of balls**

$$\delta = 2 \times 0.76 \sqrt[3]{\frac{2W^2}{(zk)^2 d}\left(\frac{1}{E_1} + \frac{1}{E_2}\right)^2}$$

Determine the Major Parameters: Thrust Bearing

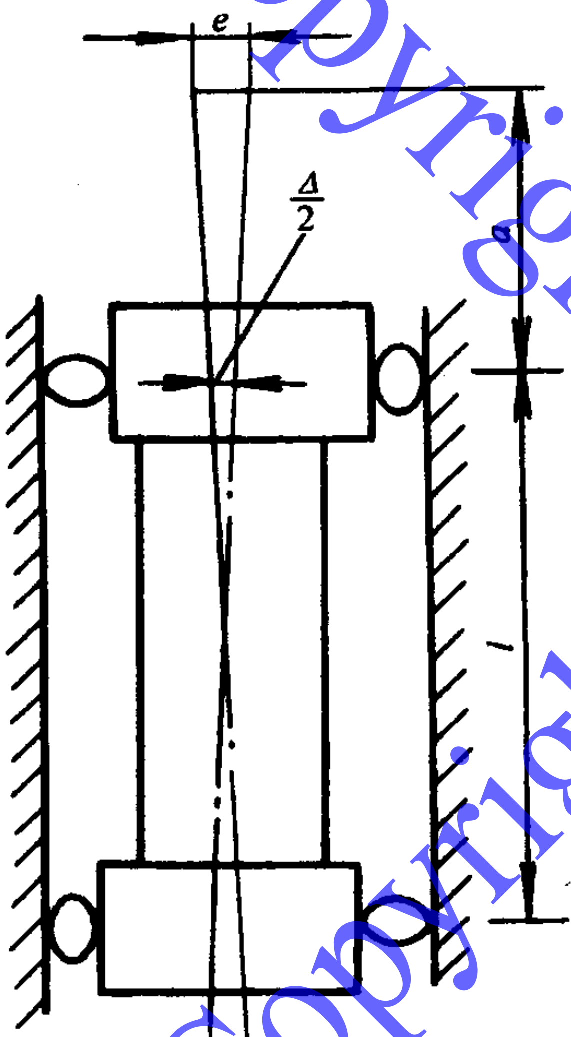


- ❒ Determine the diameter of balls
- ❒ Determine the number of balls
  - **Axial**

$$z = \frac{180^\circ}{\arcsin\frac{t}{D_0}}$$

$$t = (1.2 \sim 1.5)d$$

Determine the Major Parameters: Radial



- ❒ Determine the diameter of balls
- ❒ Determine the number of balls
  - Axial
  - Radial

$$e = \frac{\Delta\left(\frac{l}{2} + a\right)}{l}$$

$$z = 2^m N$$

Compact Ball Bearings: Introduction

❒ **Why do we need to use compact ball bearings ?**

$$\delta = 2\times 0.76\sqrt[3]{\frac{2W^2}{(zk)^2 d}\left(\frac{1}{E_1}+\frac{1}{E_2}\right)^2}$$

狄锦如, 高精度密珠轴承的设计, 机床, **1991, (5): 44-47**

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毛书越, 密珠轴承过盈量的分析与计算, 航空精密制造技术, **1991, (2): 21-23**

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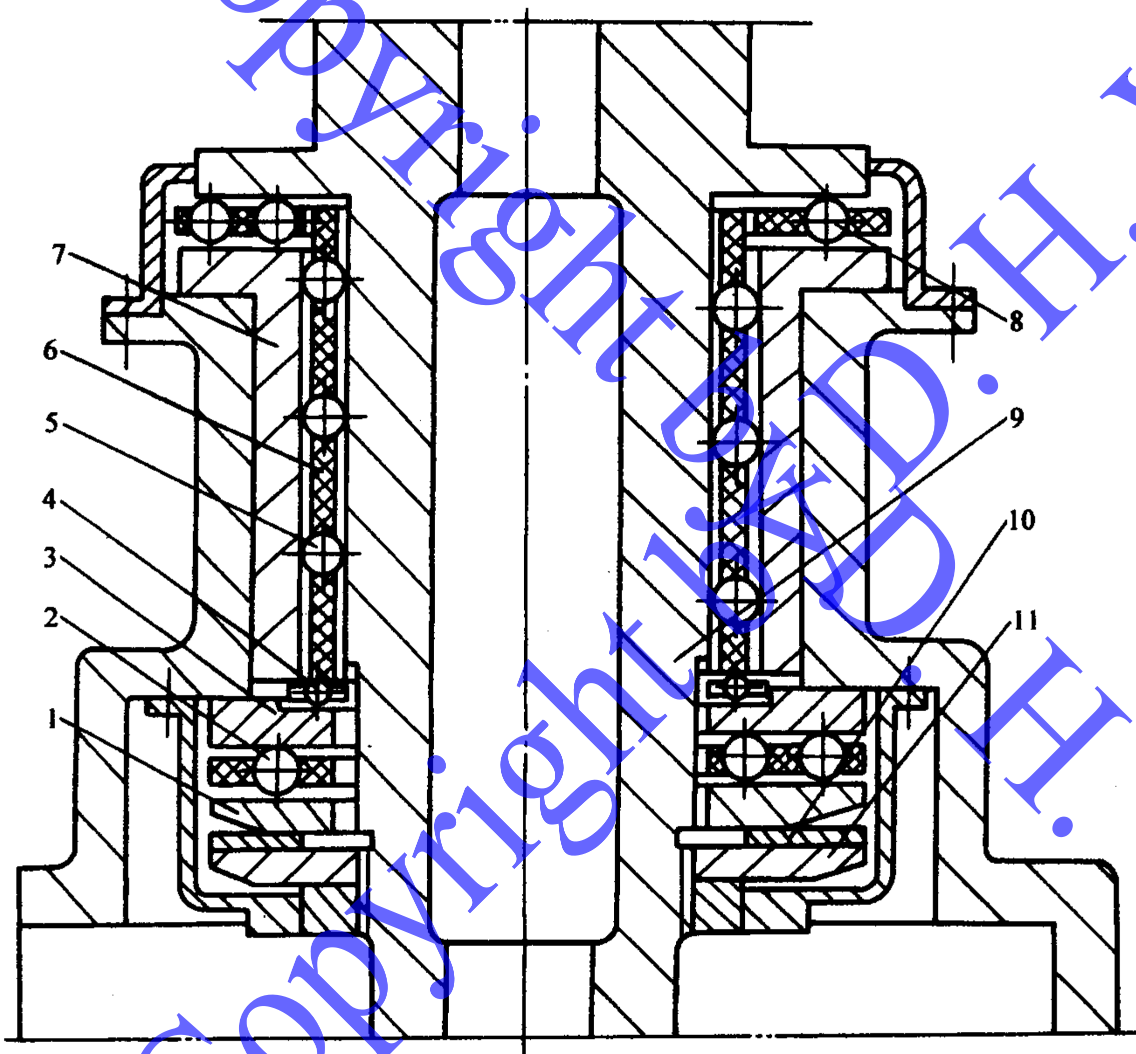
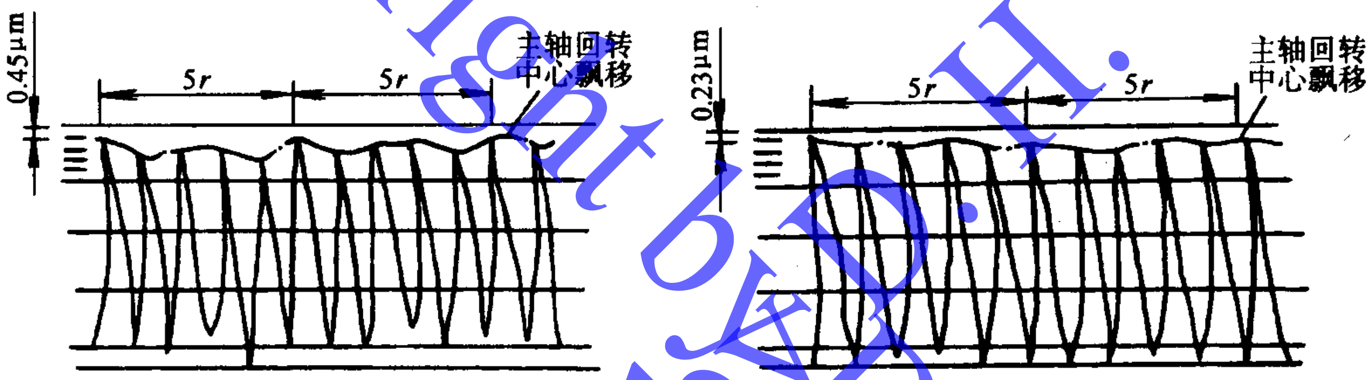
Configuration

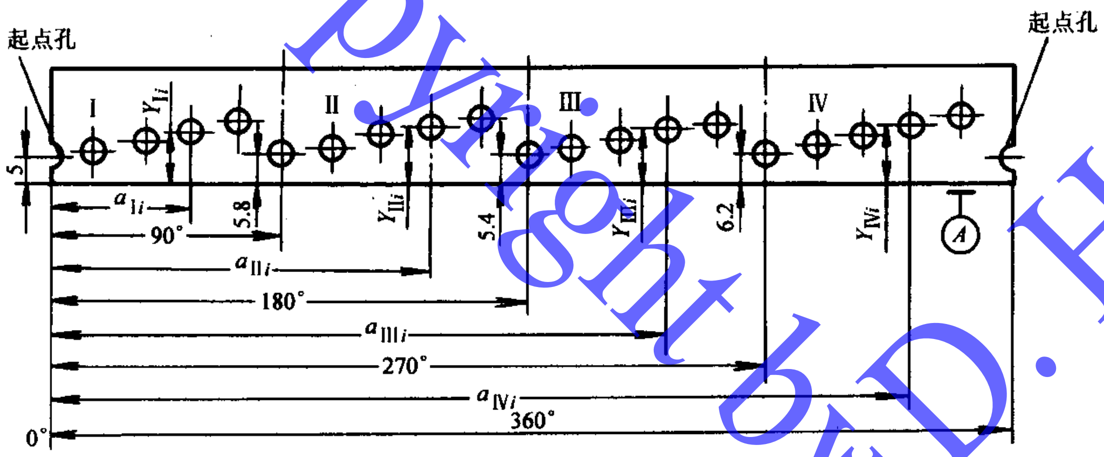


- ❒ 滚珠的密集度



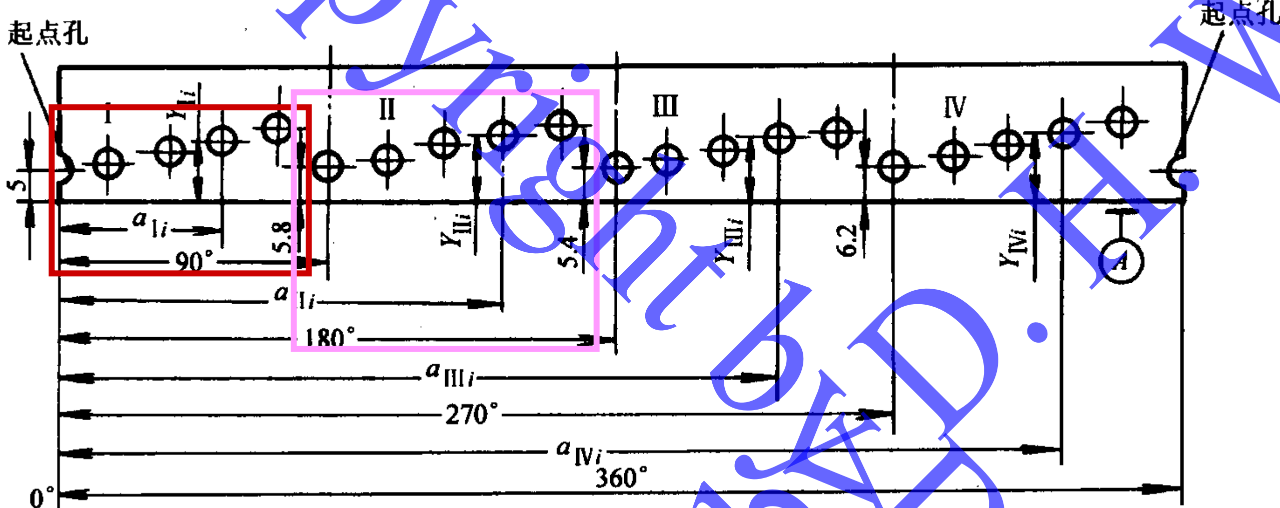
a) b)

- ❒ 滚珠的排列方式
- ❒ 过盈量的确定



- 排列原则
  - 排列方式必须满足每个滚珠的滚道互不重叠，并在直径方向上滚珠的配置成对称的原则。
- Ex：1”数字式光栅分度头径向轴承的滚珠排列

Compact Ball Bearings: Radial Bearings



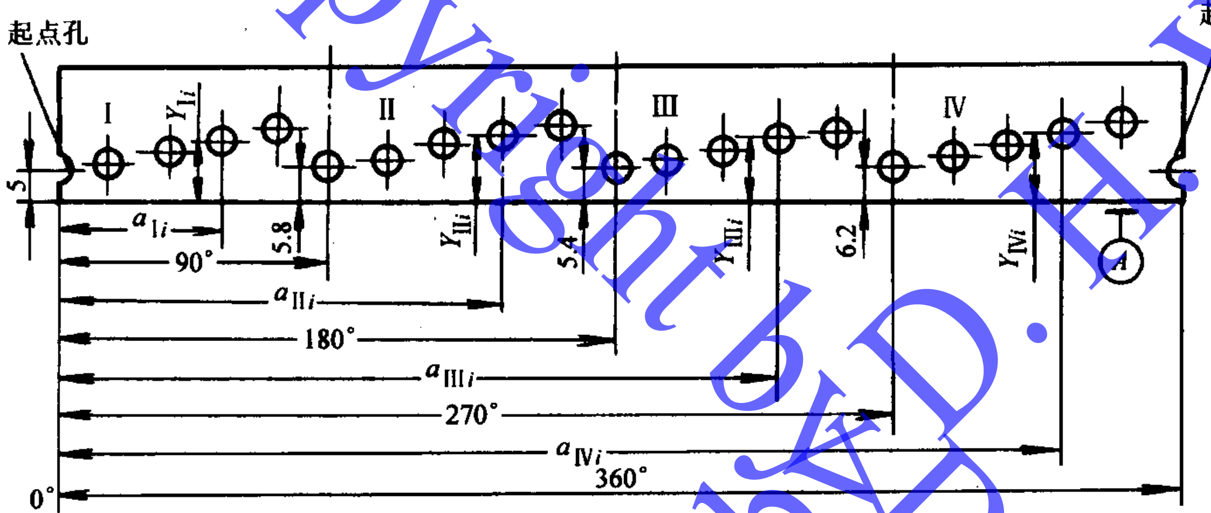
第 *I* 象限

$$j=0\begin{cases}\alpha_{Ii}=\beta j+\alpha i\\ Y_{Ii}=(l+j\Delta)+ki\end{cases}$$

第 *II* 象限

$$j=1\begin{cases}\alpha_{IIi}=\beta j+\alpha i=90^\circ+18^\circ i\\ Y_{IIi}=[l+(j+i)\Delta]+ki=5.8+1.6i\end{cases}$$

Compact Ball Bearings: Radial Bearings



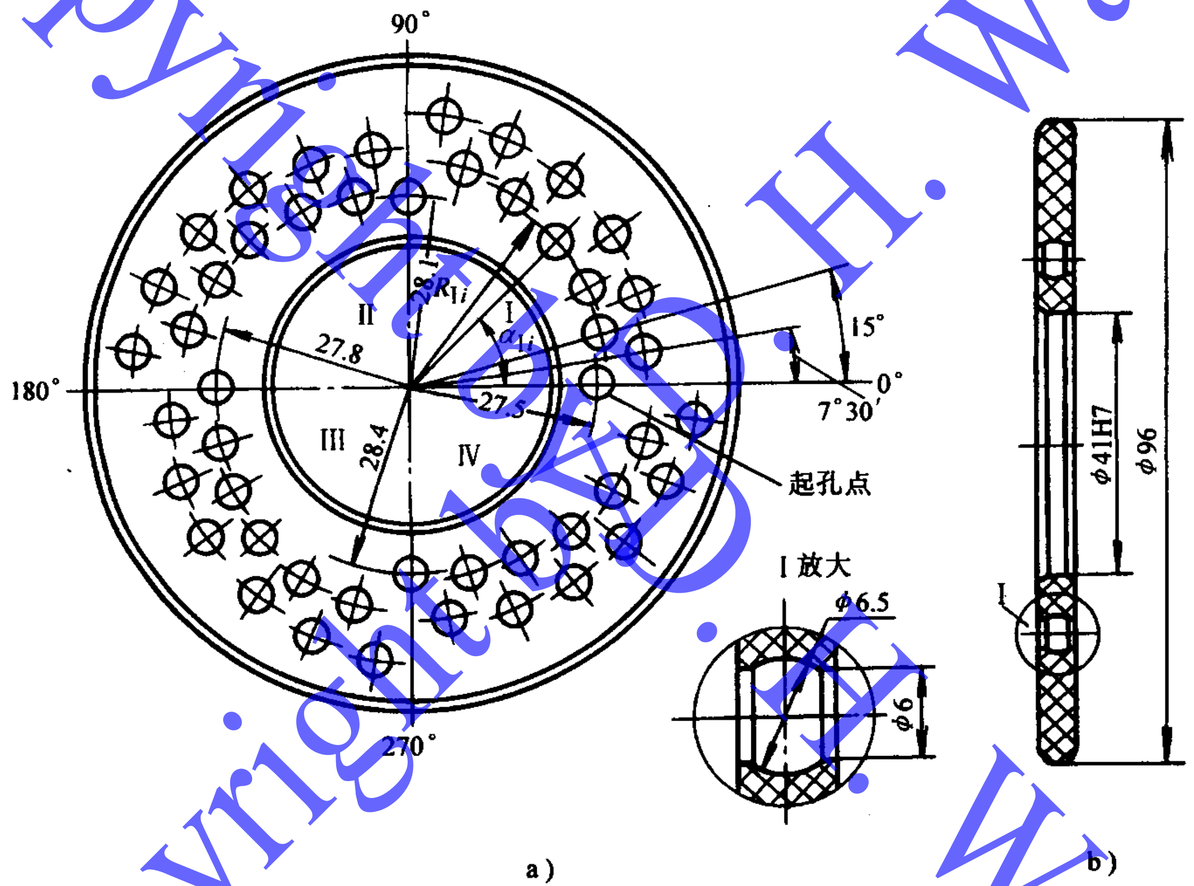
第 *III* 象限

$$j=2\begin{cases}\alpha_{IIIi}=\beta j+\alpha i=180^\circ+18^\circ i\\ Y_{IIIi}=\left[l+(j-1)\Delta\right]+ki=5.4+1.6i\end{cases}$$

第 *IIII* 象限

$$j=3\begin{cases}\alpha_{IIIIi}=\beta j+\alpha i=270^\circ+18^\circ i\\ Y_{IIIIi}=\left[l+j\Delta\right]+ki=6.2+1.6i\end{cases}$$

Compact Ball Bearings: Thrust Bearings



Compact Ball Bearings: 过盈量的确定

- 总尺寸过盈量Δ

$$\Delta = D_2 - (D_1 + 2d)$$

- 过盈量又为滚珠与轴的接触变形和滚珠与套的接触变形之和

$$\Delta = 2(\delta_1 + \delta_2) = 4\delta$$

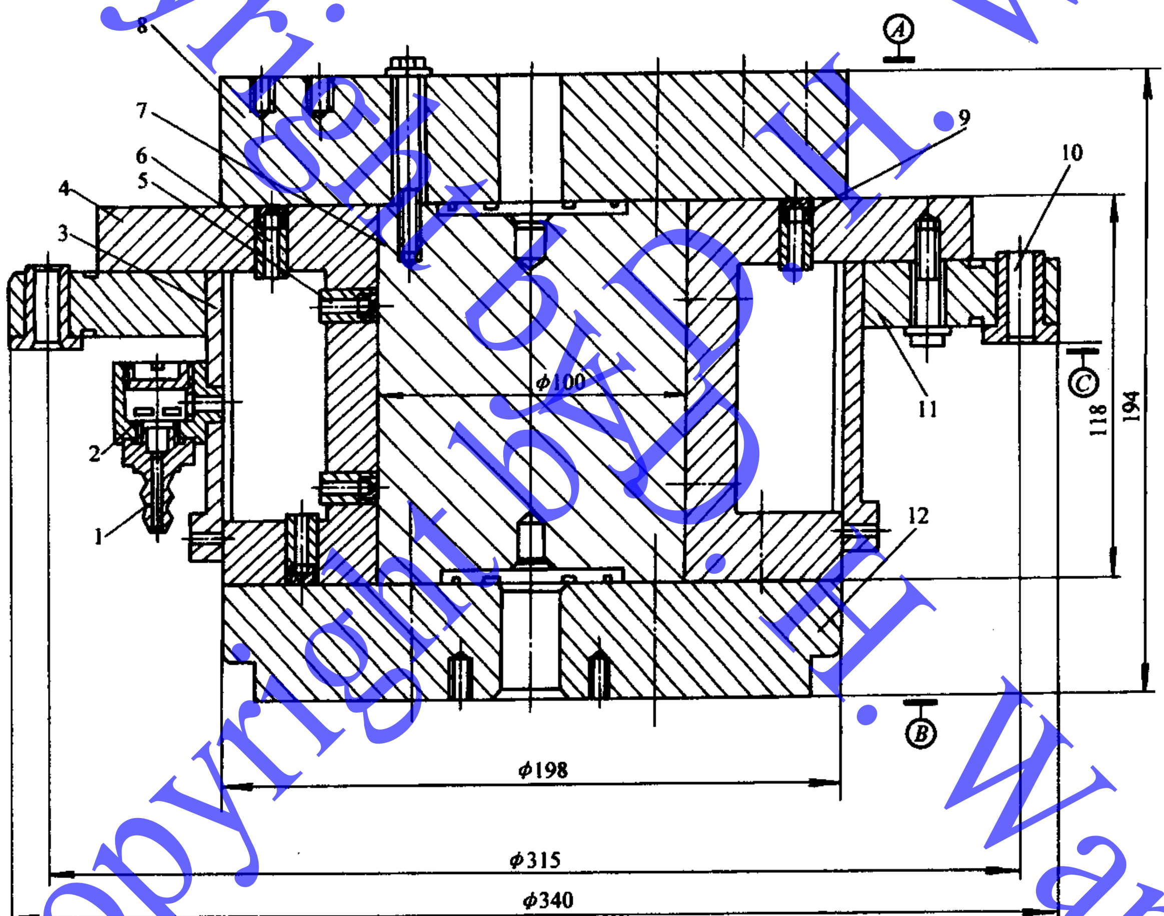
- 变形量δ的计算公式：
  - 对于径向轴承 $\delta = 1.23\left(\frac{P^2}{E^2}\frac{R+r}{Rr}\right)^{1/3}$
  - 对端面轴承： $\delta = 1.23\left(\frac{P^2}{E^2 r}\right)^{1/3}$

Aerostatic Spindles: Advantages/Disadvantages

- ❐ 气体静压轴承是以一定恒压的净化空气充满轴承套与轴之间，并以其为润滑介质构成的轴承。
- ❐ 由于空气粘性极小，因此几乎没有摩擦力矩，旋转灵活，正常工作中不磨损机件，工作中不用维修调整，具有可以在特殊环境下使用的优点。
- ❐ 由于气膜对轴承零件的加工误差有平均作用，构成轴系后的回转精度可比轴承副零件精度有很大提高。
- ❐ 目前空气静压轴承的回转精度已高达**0.01 um**，因此常用作为高精度仪器的轴承。
- ❐ 其缺点是需要无油、无水、无尘的气源，即需有较复杂的辅助设备，因此使用成本高，而且轴承的刚度一般不如机械轴承，采取了提高刚度的技术措施后，也常用在精密机床的轴系中。

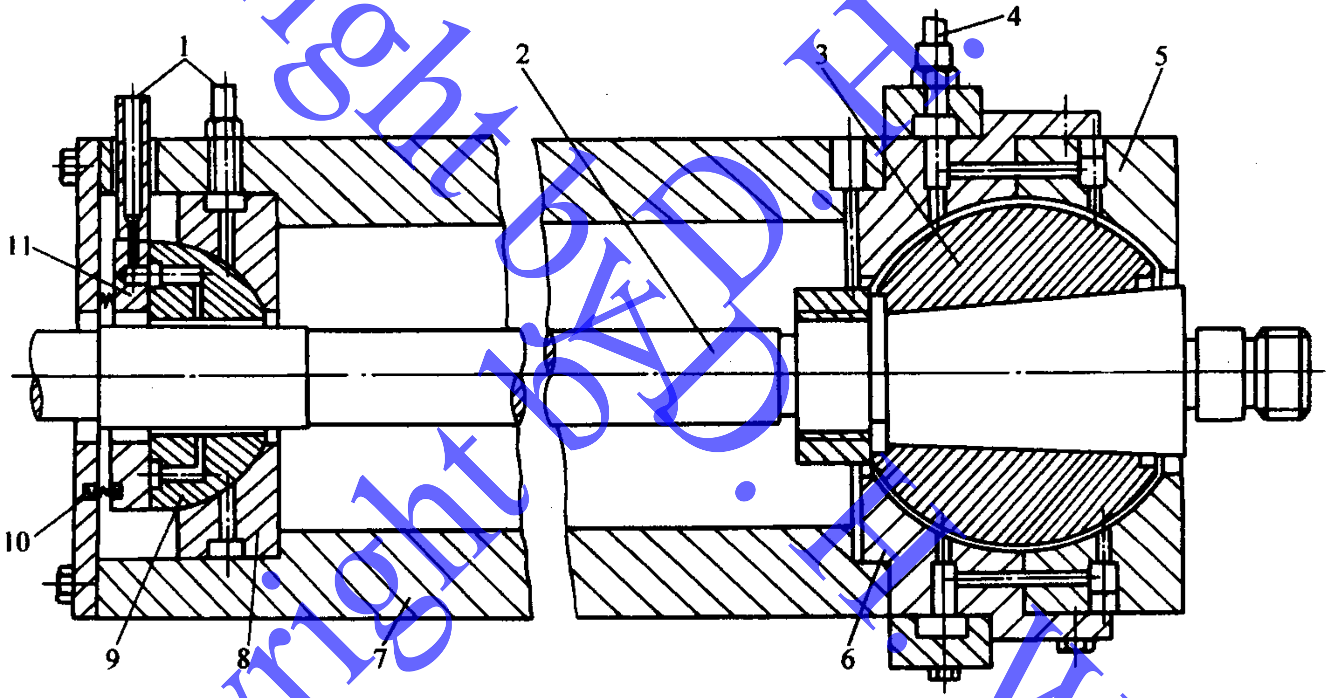
Aerostatic Bearings: Configuration

❒ 圆柱型空气静压轴承轴系



Aerostatic Bearings: Configuration

❒ 圆球与圆柱混合型空气静压轴承轴系



Hydrostatic Spindles: Advantages/Disadvantages

- Compared to rolling element spindles, hydrostatic spindles have:
  - Much lower static and dynamic run-out.
  - Usually equal or better static stiffness at the tool-point.
  - Much higher damping, higher dynamic stiffness.
  - Higher tolerance to impact loads.
  - No wear, potentially infinite life.
  - Enables machining forces to be measured by measuring pocket pressures.
  - …

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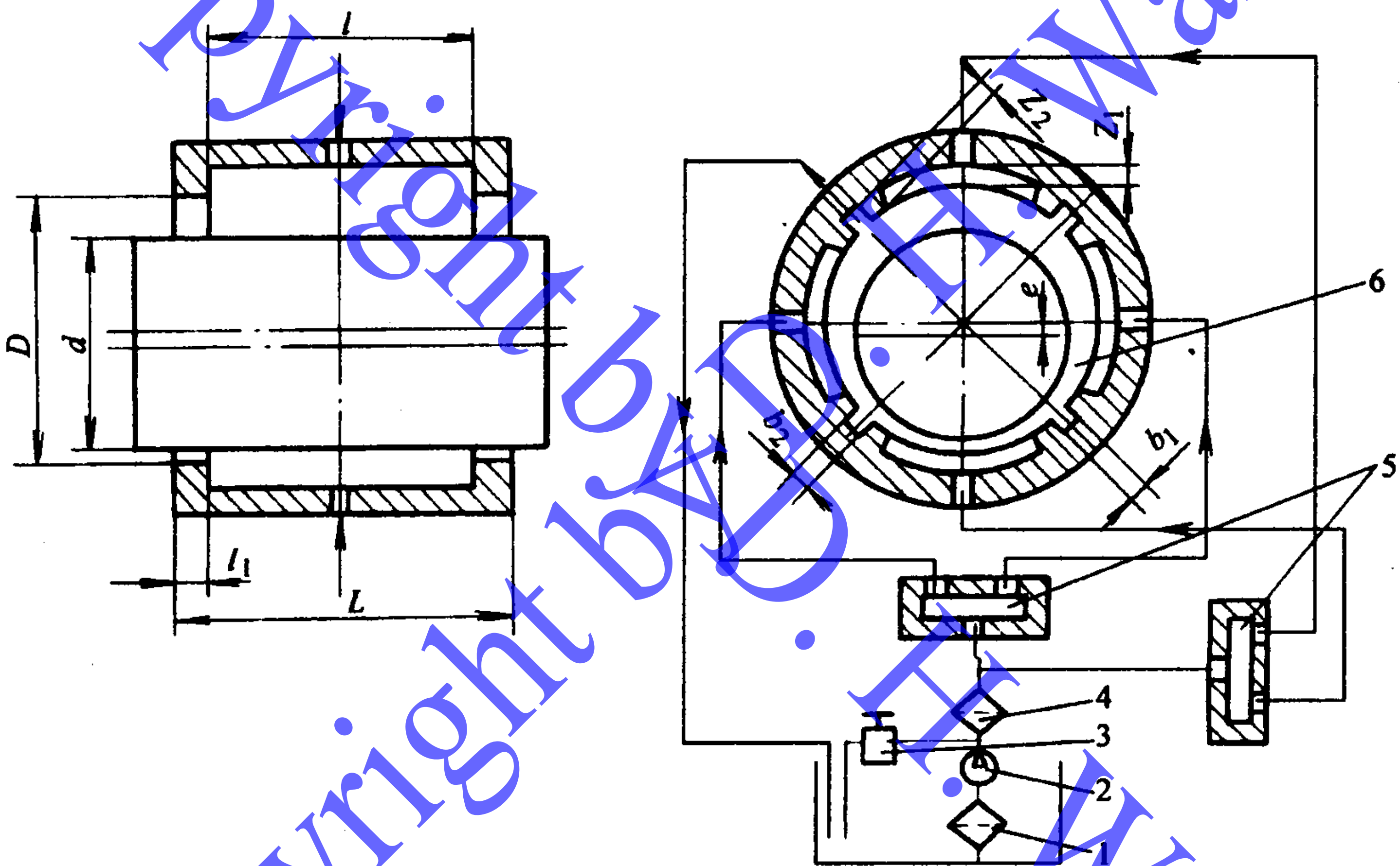
Hydrostatic Spindles: Advantages/Disadvantages

- Compared to rolling element spindles, hydrostatic spindles have:
  - …
  - Much higher power consumption, heat generation, temperature rise at high speeds (if oil is used).
  - Typically higher cost, mostly due to the required support equipment.

Hytrostatic Bearings: Operating Principle



Hytrostatic Bearings: Operating Principle

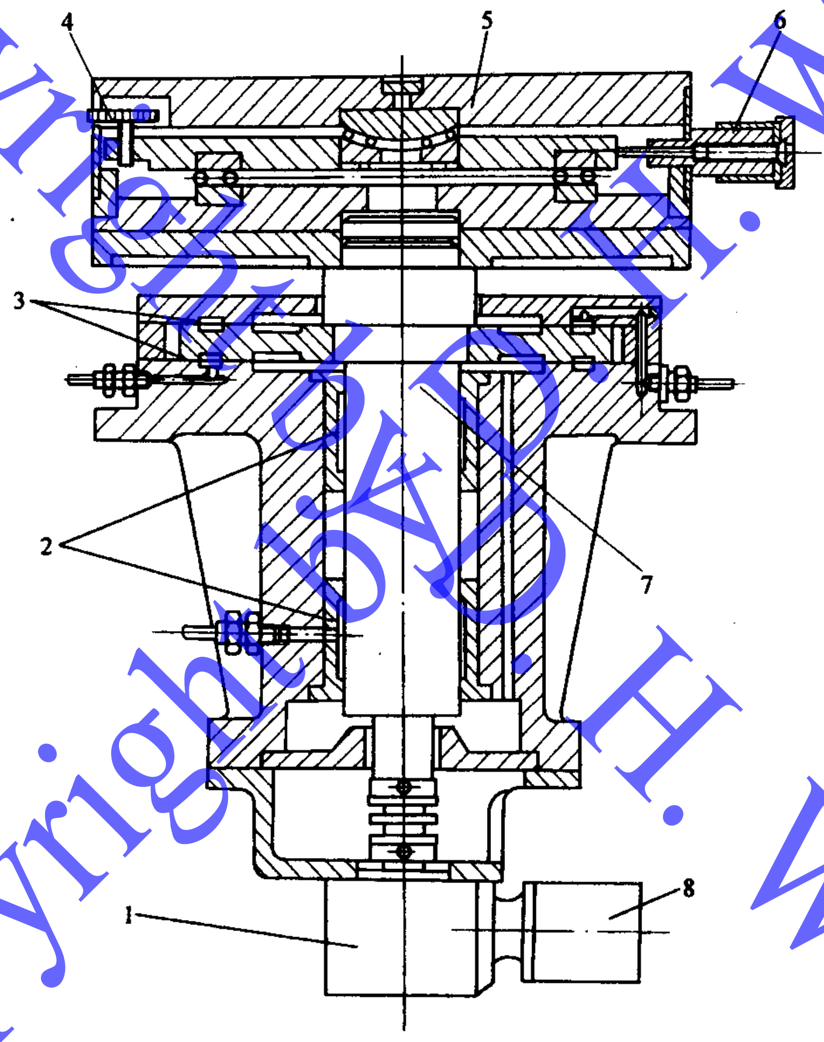
- 液体静压轴承的主要参数
  - 油腔形式和数量
  - 轴承内径D
  - 轴承的长度L
  - 轴向封油面宽度l1和周向封油边宽度b1
  - 主轴和轴承配合直径间隙2h0 推荐在
    - D＜50mm时，2h0＝(0.0006 ~ 0.01D)；
    - D为50 ~ 100mm时，2h0＝(0.0005 ~ 0.0008)D；
    - D为100 ~ 200mm时，2h0＝(0.0004 ~ 0.0007)D。
  - 油腔深度Z1 推荐Z1＝(30 ~ 60)h0。
  - …

Hytrostatic Bearings: Operating Principle

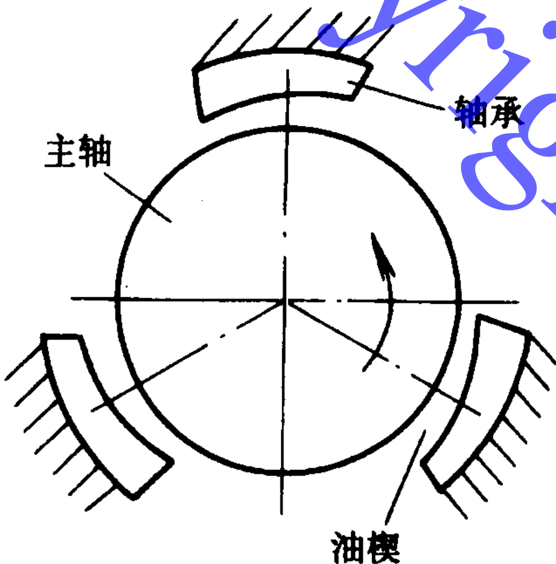
❒ 液体静压轴承的主要参数

- …
- 轴承跨距l　一般取(4 ~ 6)D。
- 回油槽深度Z2和宽度b2
- 轴与轴承的几何形状误差
- 轴承材料: HT200~400, ZHMn58-2-2, ZQSn6-6-3

Hytrostatic Bearings: Configuration

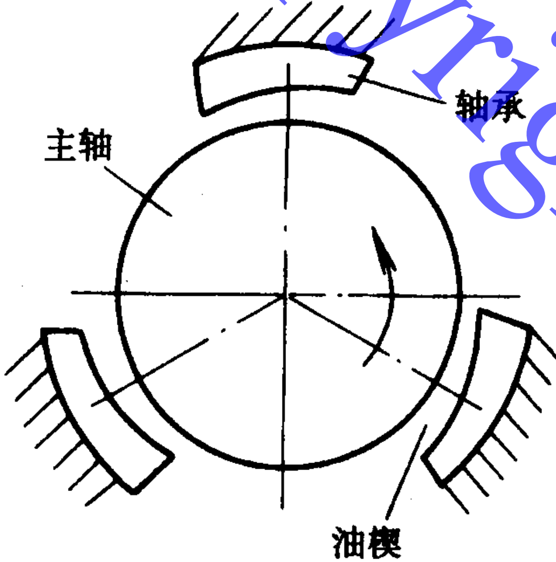


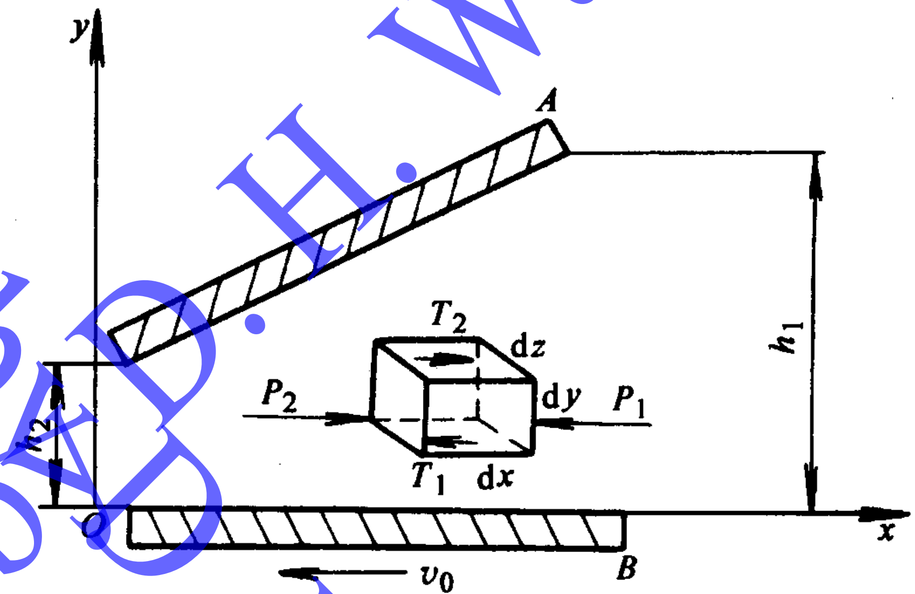
Hydrodynamic Bearings: Operating Principle



- Hydrodynamic bearings operate based on the principle that
  - **Viscous fluids are dragged between bodies as they move past each other, so the rotating shaft acts like a pump.**
  - **The pressure gradient is limited, and so is the load capacity and stiffness.**
  - **They are the simplest of bearings.**

Hydrodynamic Bearings: Operating Principle





$$p_1 = p\,dz\,dy,\; p_2 = (p + \Delta p)\,dz\,dy$$

$$T_2 = \tau\,dz\,dx,\; T_1 = (\tau + \Delta\tau)\,dz\,dx$$

$$\boxed{p_2 + T_2 = p_1 + T_1} \Rightarrow \Delta p\,dz\,dy = \Delta\tau\,dz\,dx$$

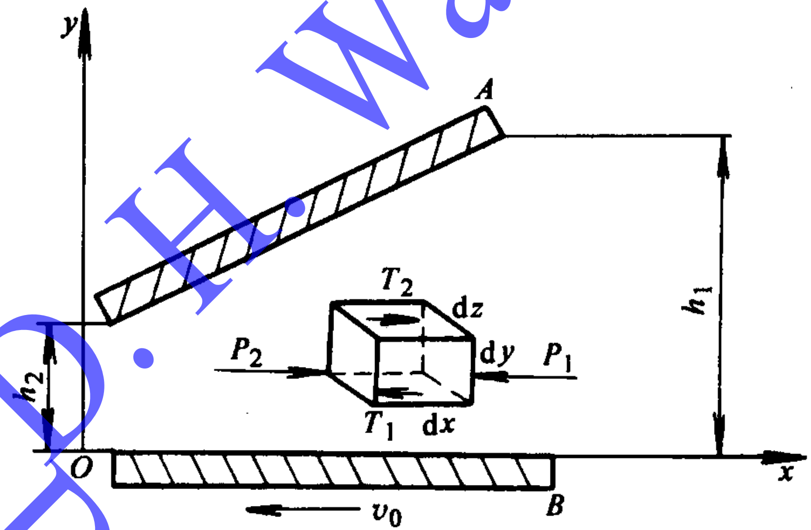
Hydrodynamic Bearings: Operating Principle

$$\Delta p \, dz \, dy = \Delta \tau \, dz \, dx$$

$$\frac{dp}{dx} = \frac{d\tau}{dy} \qquad \tau = -\nu \frac{dv}{dy}$$

$$\frac{dp}{dx} = \frac{d\tau}{dy} = -\nu \frac{d^2 v}{dy^2}$$

$$v = A \left(dp/dx \right)$$



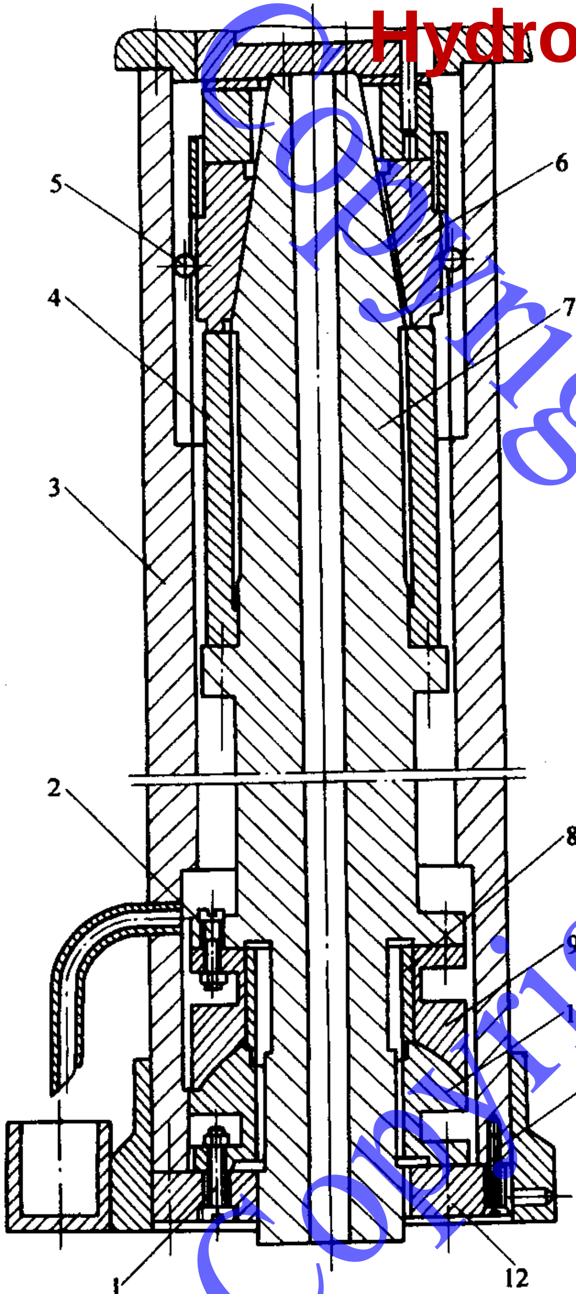
- ❐ If $h_1 = h_2$, then dv/dy is a constant.
- ❐ If $h_1 > h_2$, then …
- ❐ If the shaft doesn't rotate, $dv/dy = 0$

Hydrodynamic Bearings: Operating Principle

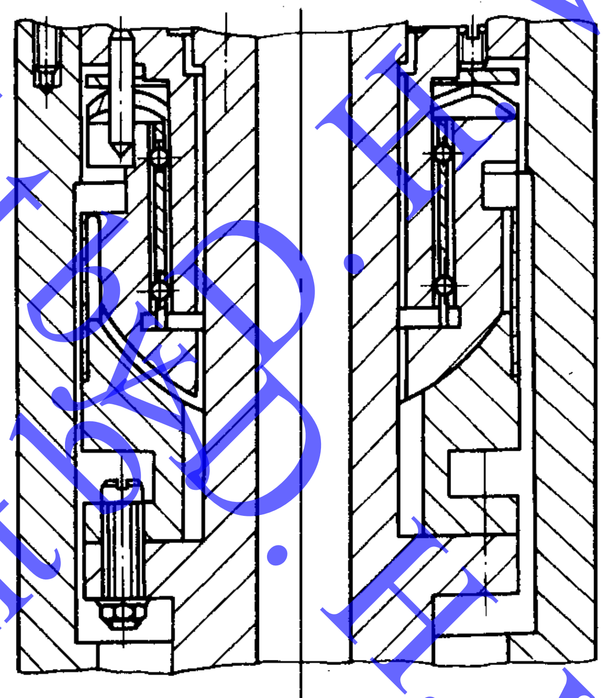
❒ 动压轴承获得油动压的条件：

- 在结构上，轴承必须有斜楔。主轴只有向斜楔减小方向转动时，才会产生油动压，若主轴转动速度高，则油膜加厚。因此为保证主轴有高的回转精度，转速应均匀。不允许向斜楔增大方向转动，这时没有油动压，主轴与轴承刚性接触，而产生磨损。
- 轴系在转动之前必须加有一定粘度的润滑油，进行充分润滑。润滑油不能随便代替，必须用圆度仪主轴专用油。

Hydrodynamic Bearings: Configuration



a)



b)

- 英国R. T. H.公司圆度仪轴系，图a)
- HYQ14型公司圆度仪轴系，图b)

Acknowledgement

Thank you very much for your attention !